CHAPTER 1

1.1 (a) $\rho = \frac{p}{RT} = \frac{1.9 \times 10^4}{(287)(203)} = \boxed{0.326 \text{ kg/m}^3}$

(b) $T = \frac{p}{\rho R} = \frac{1058}{(1.23 \times 10^{-3})(1716)} = \boxed{501\ ^{\circ}\text{R}}$

1.2 $N' = - \int_{LE}^{TE} (p_u \cos\theta + \tau_u \sin\theta)\, ds_u$

$$+ \int_{LE}^{TE} (p_\ell \cos\theta - \tau_\ell \sin\theta)\, ds_\ell \qquad (1.7)$$

$ds \cos\theta = dx$

$ds \sin\theta = -dy$

Hence,

$$N' = - \int_{LE}^{TE} (p_u - p_\ell)\, dx + \int_{LE}^{TE} (\tau_u + \tau_\ell)\, dy$$

$$N' = - \int_{LE}^{TE} [(p_u - p_\infty) - (p_\ell - p_\infty)]\, dx + \int_{LE}^{TE} (\tau_u + \tau_\ell)\, dy$$

Divide by $q_\infty S = q_\infty c(1)$

$$\frac{N'}{q_\infty c} = -\frac{1}{c} \int_{LE}^{TE} \left[\left(\frac{p_u - p_\infty}{q_\infty}\right) - \left(\frac{p_\ell - p_\infty}{q_\infty}\right)\right] dx + \frac{1}{c} \int_{LE}^{TE} \left(\frac{\tau_u}{q_\infty} + \frac{\tau_\ell}{q_\infty}\right) dy$$

$$\boxed{c_n = \frac{1}{c} \int_0^c \left(c_{p_\ell} - c_{p_u}\right) dx + \frac{1}{c} \int_{LE}^{TE} \left(c_{f_u} + c_{f_\ell}\right) dy}$$

This is Eq. (1.15).

$$A' = \int_{LE}^{TE} (-p_u \sin\theta + \tau_u \cos\theta)\, ds_u$$

$$+ \int_{LE}^{TE} (p_\ell \sin\theta + \tau_\ell \cos\theta)\, ds_\ell \qquad (1.8)$$

$$A' = \int_{LE}^{TE} (p_u - p_\ell)\, dy + \int_{LE}^{TE} (\tau_u + \tau_\ell)\, dx$$

$$A' = \int_{LE}^{TE} [(p_u - p_\infty) - (p_\ell - p_\infty)]\, dy + \int_o^c (\tau_u + \tau_\ell)\, dx$$

$$\frac{A'}{q_\infty c} = \frac{1}{c}\int_{LE}^{TE} \left[\left(\frac{p_u - p_\infty}{q_\infty}\right) - \left(\frac{p_\ell - p_\infty}{q_\infty}\right)\right] dy + \frac{1}{c}\int_o^c \left(\frac{\tau_u}{q_\infty} + \frac{\tau_\ell}{q_\infty}\right) dx$$

$$\boxed{c_a = \frac{1}{c}\int_{LE}^{TE} (c_{p_u} - c_{p_\ell})\, dy + \frac{1}{c}\int_o^c (c_{f_u} - c_{f\ell})\, dx}$$

This is Eq. (1.16).

$$M'_{LE} = \int_{LE}^{TE} [(p_u \cos\theta + \tau_u \sin\theta)x - (p_u \sin\theta - \tau_u \cos\theta)y]\, ds_u$$

$$+ \int_{LE}^{TE} [-p_\ell \cos\theta + \tau_\ell \sin\theta)x + (p_\ell \sin\theta + \tau_\ell \cos\theta)y]\, ds_\ell$$

$$M'_{LE} = \int_{LE}^{TE} [p_u - p_\ell]\, x\, dx - \int_{LE}^{TE} (\tau_u + \tau_\ell)\, x\, dy$$

$$+ \int_{LE}^{TE} [p_u - p_\ell]\, y\, dy + \int_{LE}^{TE} (\tau_u + \tau_\ell)\, y\, dx$$

$$M'_{LE} = \int_{LE}^{TE} [(p_u - p_\infty) - (p_\ell - p_\infty)]\, x\, dx - \int_{LE}^{TE} (\tau_u + \tau_\ell)\, x\, dy$$

$$+ \int_{LE}^{TE} [p_u - p_\infty) - (p_\ell - p_\infty)]\, y\, dy + \int_{LE}^{TE} (\tau_u + \tau_\ell)\, y\, dx$$

Divide by $q_\infty c^2$:

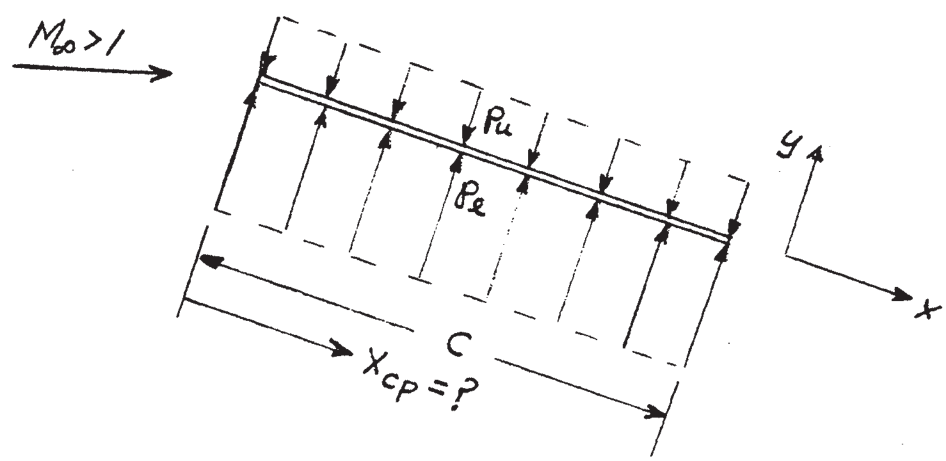
$$\frac{M_{LE'}}{q_\infty c^2} = \frac{1}{c^2}\int_{LE}^{TE} \left[\left(\frac{p_u - p_\infty}{q_\infty}\right) - \left(\frac{p_\ell - p_\infty}{q_\infty}\right)\right] x\, dx - \frac{1}{c^2}\int_{LE}^{TE} \left(\frac{\tau_u}{q_\infty} + \frac{\tau_\ell}{q_\infty}\right) x\, dy$$

$$+\frac{1}{c^2}\int_{LE}^{TE}\left[\left(\frac{p_u - p_\infty}{q_\infty}\right)-\left(\frac{p_\ell - p_\infty}{q_\infty}\right)\right] y\,dy + \frac{1}{c^2}\int_{LE}^{TE}\left(\frac{\tau_u}{q_\infty}+\frac{\tau_\ell}{q_\infty}\right) y\,dx$$

$$\boxed{\begin{aligned} c_{m_{\ell c}} = \frac{1}{c^2}\Big[\int_0^c (C_{p_u} - C_{p_\ell})\,x\,dx - \int_{LE}^{TE} (C_{f_u} + C_{f_\ell})\,x\,dy \\ + \int_{LE}^{TE} (C_{p_u} - C_{p_\ell})\,y\,dy + \int_0^c (C_{f_u} + C_{f_\ell})\,y\,dx\Big] \end{aligned}}$$

This is Eq. (1.17).

1.3



$$M'_{LE} = -\int_0^c (p_\ell - p_u)\,(dx)\,(1)\,x - (p_\ell - p_u)\int_0^c x\,dx$$

$$M'_{LE} = -(p_\ell - p_u)\,\frac{c^2}{2}$$

$$N' = \int_0^c (p_\ell - p_u)\,dx = (p_\ell - p_u)\,c$$

$$x_{cp} = -\frac{M'_{LE}}{N'} = -\frac{\left[-(p_\ell - p_u)\frac{c^2}{2}\right]}{(p_\ell - p_u)\ c}$$

$\boxed{x_{cp} = c/2}$

1.3 For a flat plate, $\theta = 0$ in Eqs. (1.7) – (1.11). Hence,

$$N' = \int_0^c (p_\ell - p_u)\,dx = \int_0^1 [-2 \times 10^4 (x-1)^2 + 1.19 \times 10^5]\,dx$$

$$N' = -2 \times 10^4 \left[\frac{x^3}{3} - x^2 + x\right]_0^1 + [1.19 \times 10^5 x]_0^1 = \boxed{1.12 \times 10^5\ \text{N}}$$

$$A' = \int_0^c (\tau_\ell - \tau_u)\,dx = \int_0^1 (731\ x^{-0.2} + 288\ x^{-0.2})\,dx$$

$$A' = [1274\ x^{0.8}]_0^1 = \boxed{1274\ \text{N}}$$

$$L' = N' \cos\alpha - A' \sin\alpha = 1.12 \times 10^5 \cos 10^\circ - 1274 \sin 10^\circ$$

$$= \boxed{1.105 \times 10^5 \text{N}}$$

$$D' = N' \sin\alpha + A' \cos\alpha = 1.12 \times 10^5 \sin 10^\circ + 1274 \cos\alpha$$

$$= \boxed{2.07 \times 10^4 \text{N}}$$

$$M'_{LE} = \int_0^c [p_u - p_\ell]\,x\,dx = \int_0^1 [2 \times 10^4 (x-1)^2 - 1.19 \times 10^5]\,x\,dx$$

$$+ 2 \times 10^4 \left[\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2}\right]_0^1 - [0.595 \times 10^5 x^2]_0^1 = \boxed{-5.78 \times 10^4\ \text{Nm}}$$

$$M'_{c/4} = M'_{LE} + L'\,(c/4) = -5.78 \times 10^4 + 1.105 \times 10^5\ (0.25)$$

$$= \boxed{-3.02 \times 10^4\ \text{N/m}}$$

$$x_{cp} = -\frac{M'_{LE}}{N'} = -\frac{(-5.78\ \times\ 10^4)}{1.12\ \times\ 10^5} = \boxed{0.516\ \text{m}}$$

1.5

$c = c_n \cos\alpha - c_a \sin\alpha$

$= (1.2) \cos 12^\circ - (0.3) \sin\alpha = \boxed{1.18}$

$c_d = c_n \sin\alpha + c_a \cos\alpha$

$= (1.2) \sin 12^\circ + (0.3) \cos\alpha = \boxed{0.279}$

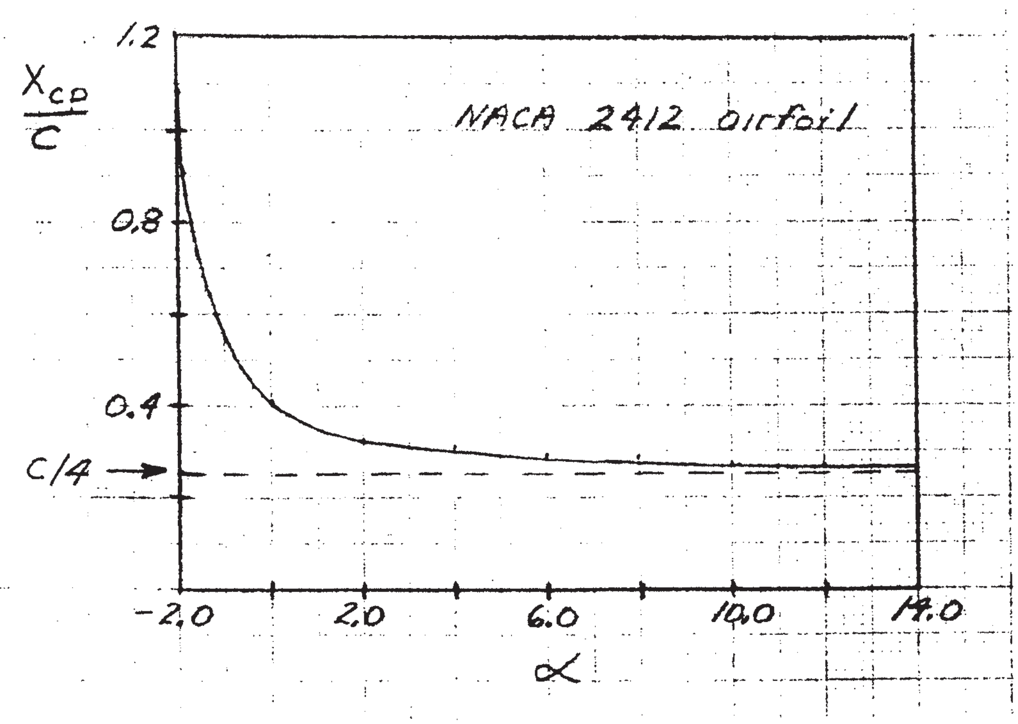
1.6 $c_n = c_\ell \cos\alpha + c_d \sin\alpha$

Also, using the more accurate N′ rather than L′ in Eq. (1.22), we have

$$x_{cp} = \frac{c}{4} - \frac{M'_{c/4}}{N'} = \frac{c}{4} - c\left(\frac{c_{m_{c/4}}}{c_n}\right)$$

<u>Hence</u>:

$\alpha(^\circ)$	c_n	x_{cp}/c
-2.0	0.0498	1.09
0	0.25	0.41
2.0	0.44	0.336
4.0	0.639	0.306
6.0	0.846	0.293
8.0	1.07	0.284
10.0	1.243	0.277
12.0	1.402	0.271
14.0	1.52	0.266



Note that x_{cp} moves forward as α is increased, and that it closely approaches the quarter-chord point in the range of α of 10° to 14°. At higher angles-of-attack, beyond the stall ($\alpha > 16^\circ$), x_{cp} will reverse its movement and move rearward as α continues to increase. Compare the above variation with the center-of-pressure measurements of the Wright Brothers on one of their airfoils, shown in Fig. 1.28.

1.7 K = 3 (mass, length, and time)

$f_1 (D, \rho_\infty, V_\infty, c, g) = 0$ Hence N = 5

We can write this expression in terms of N – K = 5 – 3 = 2 dimensionless Pi products:

$f_2 (\Pi_1, \Pi_2)$

where

$\Pi_1 = f_3 (\rho_\infty, V_\infty, c, D)$

$\Pi_2 = f_4 (\rho_\infty, V_\infty, c, g)$

Let $\Pi_1 = \rho_\infty^a \, V_\infty^b \, c^d \, D$

$1 = (m\,\ell^{-3})^a (\ell\, t^{-1})^b \,\ell^c\, (m\,\ell\, t^{-2}) = 0$

$$\left.\begin{array}{ll} \text{mass:} & a + 1 = 0 \\ \text{length:} & -3a + b + c + 1 = 0 \\ \text{time:} & -b - 2 = 0 \end{array}\right\} \qquad \begin{array}{l} a = -1 \\ b = -2 \\ c = -2 \end{array}$$

Hence:

$$\Pi_1 = \frac{D}{\rho_\infty V_\infty^{\ 2} c^2}, \text{ or } \Pi_1 = \frac{D}{\frac{1}{2}\rho_\infty V_\infty^{\ 2} c^2}$$

$$\Pi_1 = \frac{D}{q_\infty c^2}$$

Let $\Pi_2 = \rho_\infty^{\ 2}\ V_\infty\, c^b\, g^d$

$1 = (m\,\ell^{-3})^a (\ell\, t^{-1})\, \ell^b\, (\ell\, t^{-2})^d = 0$

mass: $a = 0$	$a = 0$
length: $-3a + 1 + b + d = 0$	$d = -1/2$
time: $-1 - 2d = 0$	$b = -1/2$

Hence:

$$\Pi_2 = \frac{V_\infty}{\sqrt{cg}}$$

Thus:

$$f_2\,(\Pi_1, \Pi_2) = f_2\left(\frac{D}{q_\infty c^2}, \frac{V_\infty}{\sqrt{cg}}\right) = 0$$

or:

$$\boxed{C_D = f(F_r)}$$

1.8 $D_w = f_1\,(\rho_\infty, V_\infty, c, a_\infty, c_p, c_v)$

$K = 4$ (mass, length, time, degrees)

$$f_2\ (D_w,\ \rho_\infty,\ V_\infty,\ c,\ a_\infty,\ c_p,\ c_v) = 0$$

Hence, N = 7. This can be written as a function of N – K = 7 – 4 = 3 pi products:

$$f_3 = (\Pi_1, \Pi_2, \Pi_3) = 0$$

where:

$$\Pi_1 = f_4\ (\rho_\infty,\ V_\infty,\ c,\ c_p,\ D)$$

$$\Pi_2 = f_5\ (\rho_\infty,\ V_\infty,\ c,\ c_p,\ a_\infty)$$

$$\Pi_3 = f_6\ (\rho_\infty,\ V_\infty,\ c,\ c_p,\ c_v)$$

The dimensions of c_p and c_v are

$$[c_p] = \frac{\text{energy}}{\text{mass}(^\circ)} = \frac{(\text{force})(\text{distance})}{\text{mass}(^\circ)} = \frac{(m\ell t^{-2})(\ell)}{m(^\circ)}$$

$[c_p] = \ell^2\, t^{-2}\, (^\circ)^{-1}$ where $(^\circ)$ degrees.

For Π_1:

$$\rho_\infty^{\ 1}\, V_\infty^{\ j}\, c^k\, c_p^{\ n}\, D = \Pi_1$$

$$(m\ \ell^{-3})^i\, (\ell\ t^{-1})^j\, (\ell)^k\, (\ell^2 t^{-2})^n\, (^\circ)^{-n}\, (m\ \ell\ t^{-2}) = 1$$

mass: $i + 1 = 0$ $\qquad$ $i = -1$

length: $-3i + j + k + 2n + 1 = 0$ $\qquad$ $n = 0$

time: $-j - 2n - 2 = 0$ $\qquad$ $j = -2$

degrees: $-n = 0$ $\qquad$ $k = -2$

Hence:

$$\Pi_1 = \frac{D}{\rho_\infty V_\infty^{\ 2} c^2},\ \text{or}\ \Pi_1 = \frac{D}{q_\infty c}$$

For Π_2:

$$\Pi_2 = \rho_\infty^{\ i}\, V_\infty\, c^j\, c_p^{\ k}\, a_\infty^{\ n}$$

$$1 = (m\ \ell^{-3})^i (\ell\ t^{-1}) (\ell)^j (\ell^2 t^{-2})^k (^o)^{-k} (\ell\ t^{-1})^n$$

$$\left.\begin{array}{ll} \text{mass:} & i = 0 \\ \text{length:} & -3i + 1 + j + 2k + n = 0 \\ \text{time:} & -1 - 2k - n = 0 \\ \text{degrees:} & -k = 0 \end{array}\right\} \quad \begin{array}{l} i = 0 \\ k = 0 \\ n = -1 \\ j = 0 \end{array}$$

Hence:

$$\Pi_2 = \frac{V_\infty}{a_\infty}$$

For Π_3:

$$\Pi_3 = \rho_\infty{}^i V_\infty{}^j c^k c_p{}^n c_v$$

$$1 = (m\ \ell^{-3})^i (\ell\ t^{-1})^j \ell^k (\ell^2 t^{-2})^n (^o)^{-n} (\ell\ t^{-2}) (^o)^{-1}$$

$$\left.\begin{array}{ll} \text{mass:} & i = 0 \\ \text{length:} & -3i + j + k + 2n + 2 = 0 \\ \text{time:} & -j - 2n - 2 = 0 \\ \text{degrees:} & -n - 1 = 0 \end{array}\right\} \quad \begin{array}{l} i = 0 \\ n = -1 \\ j = 0 \\ k = 0 \end{array}$$

Hence:

$\Pi_3 = \dfrac{c_v}{c_p}$. We can take the reciprocal, and still have a dimensionless product.

Hence,

$$\Pi_3 = \frac{c_v}{c_p} = \gamma$$

Thus,

$$f_3\left(\frac{D}{q_\infty S}, \frac{V_\infty}{a_\infty}, \frac{c_p}{c_v}\right)$$

or,

$\boxed{C_D = f(M_\infty, \gamma)}$

1.9 $$\frac{M_1}{M_2} = \frac{V_1}{V_2}\frac{a_2}{a_1} = \frac{V_1}{V_2}\sqrt{\frac{T_2}{T_1}} = \frac{100}{200}\sqrt{\frac{800}{200}} = 1$$

Hence, the Mach numbers of the two flows are the same.

$$\frac{Re_1}{Re_2} = \frac{\rho_1 V_1 c_1}{\rho\ V_2 c_2}\left(\frac{\mu_2}{\mu_1}\right) = \frac{\rho_1 V_1 c_1}{\rho\ V_2 c_2}\sqrt{\frac{T_2}{T_1}} = \left(\frac{1.23}{1.739}\right)\left(\frac{100}{200}\right)\left(\frac{1}{2}\right)\sqrt{\frac{800}{200}} = 0.354$$

The Reynold's numbers are <u>different</u>. Hence, the two flows are <u>not</u> dynamically similar.

1.10 Denote free flight by subscript 1, and the wind tunnel by subscript 2. For the lift and drag coefficients to be the same in both cases, the flows must be <u>dynamically similar</u>. Hence

$$M_1 = M_2$$

and

$$Re_1 = Re_2$$

For Mach number:

$$\frac{V_1}{a_1} = \frac{V_2}{a_2}$$

Since a α $\sqrt{T}$, we have

$$\frac{V_2}{\sqrt{T_2}} = \frac{V_1}{\sqrt{T_1}} = \frac{250}{\sqrt{223}} = 16.7 \qquad (1)$$

For Reynolds number: $\frac{\rho_1 V_1 c_1}{\mu_1} = \frac{\rho_2 V_2 c}{\mu_2}$

Assume, as before, that μ α $\sqrt{T}$. Hence

$$\frac{\rho_2 V_2 c_2}{\sqrt{T_2}} = \frac{\rho_1 V_1 c_1}{\sqrt{T_1}}$$

or,

$$\frac{\rho_2 V_2}{\sqrt{T_1}} = \frac{\rho_1 V_1}{\sqrt{T_1}}\left(\frac{c_1}{c_2}\right) = \frac{(0.414)(250)}{223}\left(\frac{5}{1}\right)$$

or,

$$\frac{\rho_2 V_2}{\sqrt{T_2}} = 34.65 \qquad (2)$$

Finally, from the equation of state:

$$\rho_2 T_2 = \frac{p_2}{R} = \frac{1.01 \times 10^5}{287} = 351.9 \qquad (3)$$

Eqs. (1) – (3) represent three equations for the three unknowns, ρ_2, V_2, and T_2. They are summarized below:

$$\frac{V_2}{\sqrt{T_2}} = 1.67 \qquad (1)$$

$$\frac{\rho_2 V_2}{\sqrt{T_2}} = 34.65 \qquad (2)$$

$$\rho_2 T_2 = 351.9 \qquad (3)$$

From Eq. (3):

$$\rho_2 = 351.9/T_2 \qquad (4)$$

Subst. (4) into (2):

$$\frac{351.9}{T_2}\left(\frac{V_2}{\sqrt{T_2}}\right) = 34.65 \quad (5)$$

Subst. (1) into (5): $\frac{351.9}{T_2}(16.7) = 34.65$

Hence,

$$T_2 = \frac{(351.9)(16.7)}{(34.65)} = \boxed{169.6°K}$$

From Eq. (1): $V_2 = 16.7\ \sqrt{T_2} = 16.7\ \sqrt{169.6} = \boxed{217.5\ \frac{m}{sec}}$

From Eq. (3): $\rho_2 = \frac{351.9}{T_2} = \frac{351.9}{169.6} = \boxed{2.07\frac{kg}{m^3}}$

1.11 $p_b = p_a - \rho\ g\ \Delta\ h$

$= 1.01 \times 10^5 - (1.36 \times 10^4)(9.8)(0.2)$

$p_b = \boxed{7.43 \times 10^4\ N/m^2}$

1.12 Weight = Buoyancy force + lift

$W = B + L$

$$B = \underbrace{(15{,}000)}_{\substack{\text{volume} \\ (m^3)}}\ \underbrace{(1.1117)}_{\substack{\text{air density} \\ \text{at 1000m} \\ (kg/m^3)}}\ \underbrace{(9.8)}_{\substack{\text{acceleration} \\ \text{of gravity} \\ (m/sec)}} = 1.634 \times 10^5 N$$

$$q_\infty = \frac{1}{2}\ \rho_\infty\ V_\infty{}^2 = \frac{1}{2}\ (1.1117)\ (30)^2 = 500\ N/m^2$$

$$S = \pi d^2/4 = \pi(14)^2/4 = 153.9\ m^2$$

$$L = q_\infty\ S\ C_L = (500)(153.9)(0.05) = 3487\ N$$

Hence:

$$W = 1.634 \times 10^5 + 3847 = \boxed{1.67 \times 10^5 N}$$

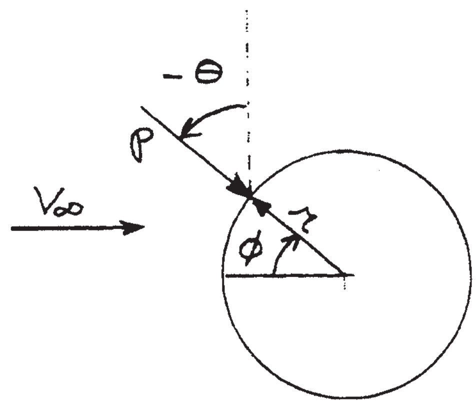
1.13 Let us use the formalism surrounding Eq. (1.16) in the text. In this case, $c_d = c_a$, and from Eq. (1.16), neglecting skin friction

$$c_d = \frac{1}{c} \int_{LE}^{TE} \left(C_{p_u} - C_{p_\ell}\right) dy \tag{1}$$

From Eq. (1.13) in the text, Eq. (1) above can be written as

$$c_d = \frac{1}{c} \int_{LE}^{TE} \left(C_{p_u} - C_{p_\ell}\right) (-\sin\theta \, ds) \tag{2}$$

Draw a picture:



Following our sign convention, note that θ is drawn counterclockwise in this sketch, hence it is a <u>negative</u> angle, -θ.

From the geometry:

$$-\theta = \pi - \phi$$

Hence, sin (-θ) = - sin θ = sin (π-θ) = cos φ

Substitute this into Eq. (2), noting also that ds = rdφ and the chord c is twice the radius, c = 2r. From Eq. (2),

$$c_d = \frac{1}{2r} \int_{LE}^{TE} \left(C_{p_u} - C_{p_\ell}\right) \cos\phi \; r \, d\phi$$

$$c_d = \frac{1}{2} \int_{LE}^{TE} \left(C_{p_u} - C_{p_\ell}\right) \cos\phi \, d\phi$$

$$c_d = \frac{1}{2} \int_{LE}^{TE} C_{p_u} \cos\phi \, d\phi - \frac{1}{2} \int_{LE}^{TE} C_{p_\ell} \cos\phi \, d\phi \tag{3}$$

Consider the limits of integration for the above integrals. The first integral is evaluated from the leading edge to the trailing edge along the <u>upper</u> surface. Hence, φ = 0 at LE and π at TE.

The second integral is evaluated from the leading edge to the trailing edge along the bottom surface. Hence, $\phi = 2\pi$ at LE and π at the TE. Thus, Eq. (3) becomes

$$c_d = \frac{1}{2}\int_0^{\pi} C_{p_u} \cos\phi \, d\phi \quad - \frac{1}{2}\int_{2\pi}^{\pi} C_{p_\ell} \cos\phi \, d\phi \qquad (4)$$

In Eq. (4),

$C_{p_u} = 2\cos^2\phi$ for $0 \le \phi \le \pi/2$

$C_{p_u} = 0$ for $\frac{\pi}{2} \le \phi \le \pi$

$C_{p_\ell} = 2\cos^2\phi$ for $\frac{3\pi}{2} \le \phi \le 2\pi$

$C_{p_\ell} = 0$ for $\pi \le \phi \le \frac{3\pi}{2}$

Thus, Eq. (4) becomes

$$c_d = \int_0^{\pi/2} \cos^3\phi \, d\phi - \int_{2\pi}^{3\pi/2} \cos^3\phi \, d\phi$$

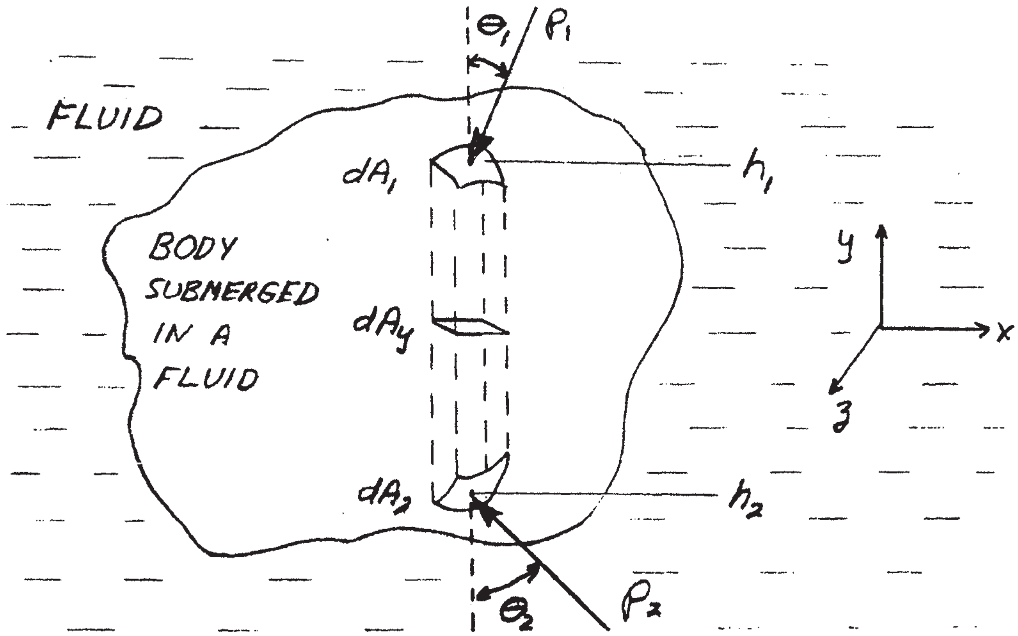
Since $\cos^3\phi \, d\phi = (\frac{1}{3}\sin\phi)(\cos^2\phi + 2)$, Eq. (5) becomes

$$c_d = [(\frac{1}{3}\sin\phi)(\cos^2\phi + 2)]_0^{\pi/2} - [(\frac{1}{3}\sin\phi)(\cos^2\phi + 2]_{2\pi}^{3\pi/2}$$

$$c_d = (\frac{1}{3})(1)(2) - (\frac{1}{3})(-1)(2)$$

$$\boxed{c_d = 4/3}$$

1.14



Consider the arbitrary body sketched above. Consider also the vertical cylinder element inside the body which intercepts the surface area dA_1 near the top of the body, and dA_2 near the bottom of the body. The pressures on dA_1 and dA_2 are p_1 and p_2 respectively, and makes angles θ_1 and θ_2 respectively with respect to the vertical line through the middle of dA_1 and dA_2. The net pressure force in the y-direction on this cylinder is:

$$dF_y = - p_1 \cos\theta_1 \, dA_1 + p_2 \cos\theta_2 \, dA_2 \tag{1}$$

Let dA_y be the projection of dA_1 and dA_2 on a plane perpendicular to the y axis.

$$dA_y = \cos\theta_1 \, dA_1 = \cos\theta_2 \, dA_2$$

Thus, Eq. (1) becomes

$$dF_y = (p_2 - p_1) \, dA_y \tag{2}$$

From the hydrostatic equation

$$p_2 - p_1 = \int_{h_1}^{h_2} \rho \, g \, dy \tag{3}$$

Combining Eqs. (2) and (3),

$$dF_y = \int_{h_1}^{h_2} \rho \, g \, dy \, dA_y \tag{4}$$

However, $dy \, dA_y = dV$ = element of volume of the body. Thus, the total force in the y direction, F_y, is given by Eq. (4) integrated over the volume of the body

$$\underbrace{F_y}_{\text{Force on body}} = \underbrace{\oiiint_V \rho g \, dV}_{\text{Weight of fluid displaced by body.}}$$

1.15 From Eq. (1.45)

$$C_L = \frac{L}{q_\infty S} = \frac{2W}{\rho_\infty V_\infty^2 S} = \frac{2(2950)}{(0.002377)V_\infty^2(174)}$$

$$C_L = \frac{14265}{V_\infty^2} \qquad (1)$$

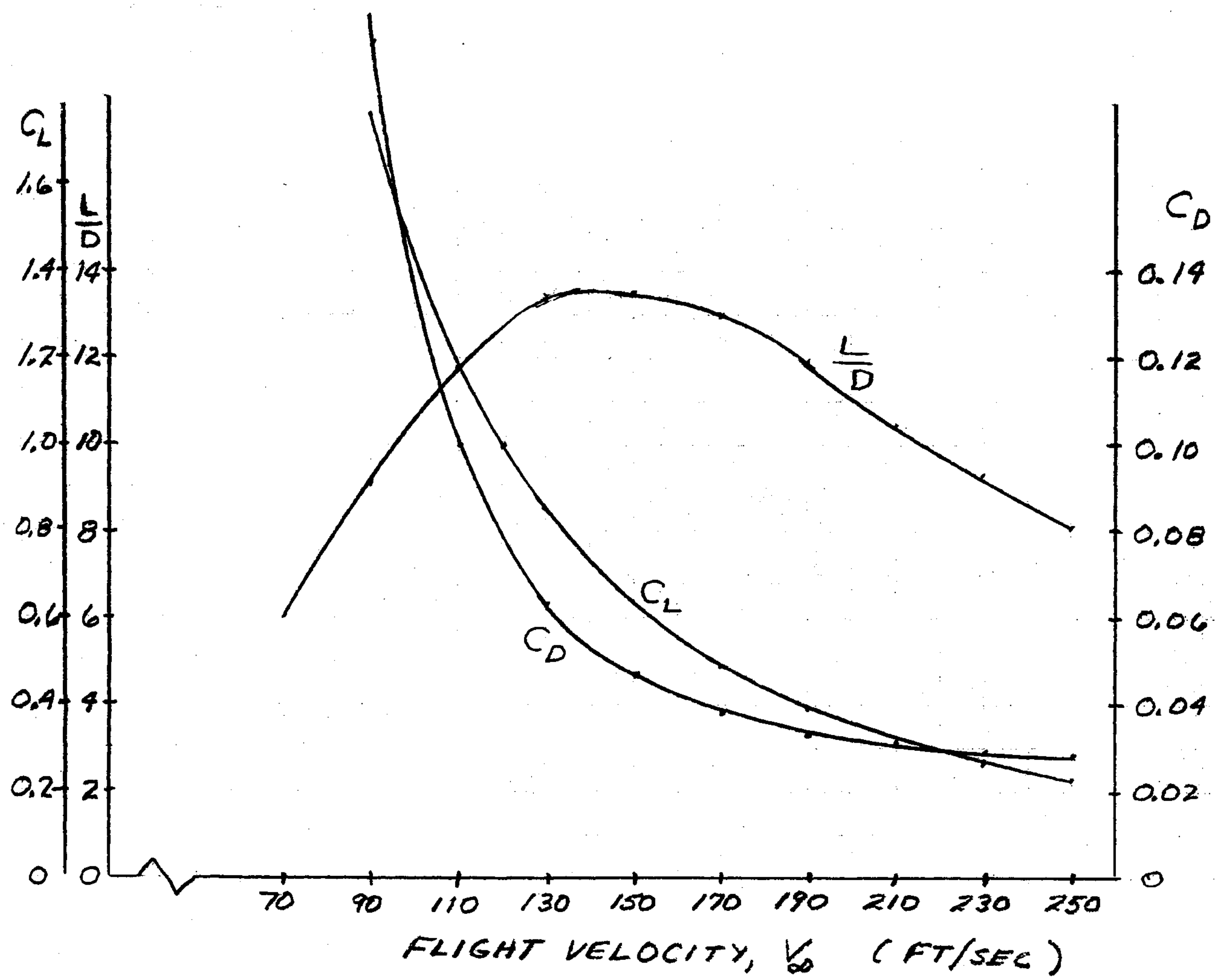
Also,

$$C_D = 0.025 + 0.054\, C_L^2 \qquad (2)$$

Tabulate Eqs. (1) and (2) versus velocity.

V_∞ (ft/sec)	C_L	C_D	$\frac{L}{D} = \frac{C_L}{C_D}$
70	2.911	0.483	6.03
90	1.761	0.192	9.17
110	1.179	0.100	11.79
130	0.844	0.063	13.40
150	0.634	0.047	13.49
170	0.494	0.038	13.0
190	0.395	0.033	11.97
210	0.323	0.031	10.42
230	0.270	0.029	9.31
250	0.228	0.028	8.14

These results are plotted on the next page.



Examining this graph, we note, for steady, level flight:

1. The lift coefficient decreases as V_∞ increases.
2. At lower velocity range, the drag coefficient decreases even faster than the lift coefficient with velocity. (Note that on the graph the scale for C_D is one-tenth that for C_L.)
3. As a result, the lift-to-drag ratio first increases, goes through a maximum, and then gradually decreases as velocity increases.

It can be shown that the maximum velocity for this airplane is about 265 ft/sec at sea level. As seen in the graph, the maximum value of L/D occurs around V_∞ = 140 ft/sec, which is much lower than the maximum velocity. However, at higher velocity the value of L/D decreases only gradually as V_∞ increases. This has the practical implication that at higher speeds, even though the value of L/D is less than its maximum, it is still a reasonably high value. The range of the aircraft is proportional to L/D (see for example, Anderson, Aircraft Performance and Design, McGraw-Hill, 1999, or Anderson, Introduction to Flight, 4th ed., McGraw-Hill, 2000). To obtain maximum range, the airplane should fly at the velocity for maximum L/D, which for this case is 140 ft/sec. However, one reason to fly in an airplane is to get from one place to another in a reasonably short time. By flying at the low velocity of $V_\infty = 140$ ft/sec, the flight time may be unacceptably long. By cruising at a higher speed, say 200 ft/sec, the flight time will be cut by 30%, with only an 18% decrease in L/D.

1.16 From Eq. (1.59), in the text,

$$\tau_w = \mu \left(\frac{dV}{dy}\right)_{y=0}$$

or,

$$\left(\frac{dV}{dy}\right)_{y=0} = \frac{\tau_w}{\mu}$$

As given in Section 1.11, μ at standard sea level temperature is

$$\mu = 1.7894 \times 10^{-5} \text{ kg/(m)(s)}$$

Thus,

$$\left(\frac{dV}{dy}\right)_{y=0} = \frac{\tau_w}{\mu} = \frac{282}{1.7894 \times 10^{-5}} = \boxed{1.576 \times 10^{7} \ sec^{-1}}$$

1.17 From Eq. (1.60)

$$\left(\frac{dT}{dy}\right)_{y=0} = -\frac{\dot{q}_w}{k}$$

The heat transfer <u>into</u> the surface is -0.03 MW/m^2, and from Section 1.11, the thermal conductivity at standard sea level temperature is

$k = 2.53 \times 10^{-2}$ J/(m)(s)(K)

$\dot{q}_w = -0.03$ MW/m^2 = -0.03×10^6 W/m^2

$$\left(\frac{dT}{dy}\right)_{y=0} = \frac{0.03 \times 10^6}{2.53 \times 10^{-2}} = \boxed{1.186 \times 10^6 \ \frac{K}{m}}$$

1.18 (a) At standard sea level, from Appendix E, $\rho_\infty = 0.002377$ slug/(ft^3). Also, from Section 1.11, $\mu_\infty = 3.7373 \times 10^{-7}$ slug/(ft)(s). Putting the velocity in consistent units,

$$V_\infty = (200)\,\frac{88}{60} = 293.3 \text{ ft/sec}$$

Thus,

$$Re = \frac{\rho_\infty V_\infty c}{\mu_\infty} = \frac{(0.002377)(293.3)(14.25)}{3.7373 \times 10^{-7}}$$

$$\boxed{Re = 2.66 \times 10^7}$$

(b) $$Re = \frac{\rho_\infty V_\infty c}{\mu_\infty} = \frac{(0.002377)(1340)(21.5)}{3.7373 \times 10^{-7}}$$

$$\boxed{Re = 1.83 \times 10^8}$$

1.19 From Figure 1.65, we have the normal and tangential force coefficients at 3-degres angle of attack given as 0.546 and 0 respectively. From Eqs. (1.1) and (1.2), written in terms of force coefficients, and using Lilienthal's nomenclature:

$$C_L = \eta \cos \alpha - \theta \sin \alpha$$

$$C_D = \eta \sin \alpha + \theta \cos \alpha$$

The ratio of lift-to-drag is

$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{\eta \cos \alpha - \theta \sin \alpha}{\eta \sin \alpha + \theta \cos \alpha}$$

$$\frac{L}{D} = \frac{0.546 \cos \ 3° - 0}{0.546 \sin 3° + 0} = \cot 3° = \boxed{19.08}$$

At $\alpha = 2^o$, from the Lilienthal Table, $\eta = 0.489$ and $\theta = 0.008$

$$\frac{L}{D} = \frac{0.489 \cos 2° - 0.008 \sin 2°}{0.489 \sin 2° + 0.008 \cos 2°} = \frac{0.4887 - 2.79 \times 10^{-4}}{0.0171 + 8 \times 10^{-3}}$$

$$\frac{L}{D} = \frac{0.488}{0.0251} = 19.44$$

At $\alpha = 1^o$, $\eta = 0.434$ and $\theta = 0.016$

$$\frac{L}{D} = \frac{0.434 \cos 1° - 0.016 \sin 1°}{0.434 \sin 1° + 0.016 \cos 1°}$$

$$\frac{L}{D} = \frac{0.4339 - 2.792 \times 10^{-4}}{7.574 \times 10^{-3} + 0.01599} = \frac{0.4336}{0.023356} = 18.56$$

At $\alpha = 4^o$, $\eta = 0.6$ and $\theta = 0.007$

$$\frac{L}{D} = \frac{0.6\cos 4^\circ + 0.007\sin 4^\circ}{0.6\sin 4^\circ + 0.007\cos 4^\circ} = \frac{0.5985 + 4.883 \times 10^{-4}}{0.04185 - 6.9829 \times 10^{-3}}$$

$$\frac{L}{D} = \frac{0.59899}{0.03487} = 17.18$$

So we have the following tabulation:

α-degrees	L/D
1^o	18.56
2^o	19.44
3^o	19.08
4^o	17.18

The Wright brothers chose a three-degree angle of attack for their design point because, from the Lilienthal Table, it corresponded very nearly to the maximum, L/D.