# **Chapter 1** Introduction to Derivatives

# Question 1.1

#### We will look at the CME

- 1. The CME trades derivatives (specifically futures and options on futures) on a wide variety of assets and indices/variables. Assets include basic commodities such as Cattle, Hogs, Pork Bellies, Lumber, Milk, and Butter. Derivatives are also traded on weather (various measures of temperature), stock indices (such as the S&P, NASDAQ, Nikkei, and Russell), interest rates (such as the Eurodollar interest rate), and currencies.
- 2. At the time of this writing, monthly trading volume is available from: http://www.cme.com/trading/dta/hist/monthly\_volume.html.

Table 1.1 summarizes the trading volume for October 2007 for the major classes of asset types.

|                    | Volume     | Month Ago  | Year Ago   | Jan-Oct, 2007 | Jan-Oct, 2006 |
|--------------------|------------|------------|------------|---------------|---------------|
| Futures            |            |            |            |               |               |
| Commodity & Others | 1,370,451  | 1,528,494  | 1,408,670  | 15,877,224    | 14,528,206    |
| Currency           | 11,832,907 | 11,710,211 | 8,723,870  | 116,996,045   | 87,876,871    |
| Equity & Index     | 59,972,745 | 55,584,345 | 38,230,166 | 516,159,044   | 387,933,512   |
| Interest Rate      | 46,012,514 | 49,812,941 | 46,537,725 | 528,881,778   | 419,626,901   |
| Options            |            |            |            |               |               |
| Commodity & Others | 223,690    | 1,90,723   | 1,587,41   | 1,661,814     | 1,536,658     |
| Currency           | 4,26,897   | 3,51,360   | 2,27,990   | 3,501,472     | 2,668,271     |
| Equity & Index     | 31,97,318  | 2,643,714  | 2,848,116  | 34,983,389    | 22,773,974    |
| Interest Rate      | 19,447,226 | 27,739,787 | 21,993,883 | 273,280,944   | 233,235,408   |

 Table 1.1
 CME Trading Volume, October 2007

3. For notional value, consider the exchange rate contracts traded on the CME as shown in Table 1.2. As we can see, contract size (i.e., number of units of the currency) for currencies with a lower US dollar value (e.g., the Japanese Yen) are larger. If the exchange chose to cut the size of the contract in half, traders would likely double their volume (all else equal) leaving notional volume constant.

Table 1.2 demonstrates that volume alone might not capture the relative economic significance of the contracts. For example, the Euro contracts have approximately 3.8 times the trading volume of Australia, but the notational value is close to 7 times.

|            |           |                      |          | <b>Contract Not1.</b> |                   |
|------------|-----------|----------------------|----------|-----------------------|-------------------|
|            | Volume    | <b>Contract Size</b> | FX (USD) | Value                 | Oct. Not1. Value  |
| AUST DLR   | 1,015,852 | 100,000              | 0.9194   | \$ 91940.00           | \$ 93,397,432,880 |
| BRIT PND   | 1,729,780 | 62,500               | 2.0633   | \$128956.25           | \$223,065,942,125 |
| CANADA DLR | 1,053,991 | 100,000              | 1.0475   | \$104750.00           | \$110,405,557,250 |
| EURO FX    | 3,831,543 | 125,000              | 1.4410   | \$180125.00           | \$690,156,682,875 |
| JAPAN YEN  | 2,493,072 | 12,500,000           | 0.0087   | \$109012.50           | \$271,776,011,400 |
| MEX PESO   | 403,697   | 500,000              | 0.0933   | \$ 46640.00           | \$ 18,828,428,080 |
| SWISS FRNC | 1,129,810 | 125,000              | 0.8590   | \$107375.00           | \$121,313,348,750 |

- 1. Trader A initially entered 20 long contracts and then reversed the 20 positions, hence having a net position of 0 contracts at the end of the day. Trader B had 5 short contracts initially, but his trade of 10 long contracts nets to 5 long contracts at the end of the day. Trader C has 25 short contracts and lowers this by 20, hence having a net position of 5 short contracts. Notice the total net position is, as it must always be for derivative contracts, zero; for every long position, there is a short position.
- 2. The four trades have a total volume of 5+15+10+20=50. At the end of the day, the open interest is 5 (Traders B and C have these offsetting positions open). The notional value of the volume is  $50 \times \$100 = \$5000$  and the notional value of the open interest is  $5 \times \$100 = \$500$ .
- 3. If the final trade had C going long 5 contracts, with B going short on the opposite side, the 5 open positions from our previous answer would be closed. Volume for the day would be 55, notional volume would be \$5500, and open interest would be zero.
- 4. If the notional value of each contract was now \$20 due to the number of widgets per contract being decreased, volume and open interest would increase five-fold and notional volume and notional open interest would be held constant. For example, imagine widgets were \$1 each and instead of 100 widgets per contract there were now 20 widgets. The first trade would have both Traders A and B wanting to enter into 25 contracts (they are interested in a trade consisting of 500 widgets and/or \$500).

If notional value of each contract was now \$20 due to the price of widgets being lower, then the answer depends on the motivation of the traders. As an example, imagine each contract is on 100 widgets, but widget prices were \$0.20 instead of \$1. If most of the traders were speculating on widget prices, we might expect the notional volume and notional open interest to be unchanged. The traders would increase their positions five-fold for they want to speculate on a certain dollar amount of widgets. For example, the first trade has A and B speculating on \$500 worth of widgets; they would now enter in 25 contracts instead of 5.

If the traders were hedging a specific number of widgets that they currently own or will be purchasing in the future, then volume and open interest might be relatively unchanged (lowering notional volume by 80%).

Heating degree-days (HDD) and cooling degree-days (CDD) are measures of abnormally cold and hot weather (respectively). High values of HDD and CDD would be times when there would be a need to heat and cool (respectively) homes and offices. For contracts traded on the Chicago Mercantile Exchange, HDD and CDD measure the summation of degrees during the month when the average daily temperature is below/above (respectively) 65°F. As of the date of this writing, December 10, 2007, the CME offers HDD and CDD contracts for 18 US cities. For HDD, there are contracts for the months April through October; for CDD, there are contracts for the months October through April. The CME also offers HDD for several European cities and HDD and CDD for Canadian cities. The CME has also introduced futures contracts for Asia-Pacific based on average temperatures.

To see how to calculate these measures, consider the following example. Assume during the month of May, the average daily temperature (average of the high and low temperature) in New York had 6 days below  $65^{\circ}$ : 48, 52, 63, 49, 45, and 38. The HDD would be 17+13+2+16+20+27=95. One can see that HDD could be very large (e.g., 1000) in Northern US cities during the winter. Notice that all days above  $65^{\circ}$  do note enter into the calculation.

Cooling degree-days work similarly, except they only consider days above 65°. Such are the days when there would be the need to use air conditioning.

This problem asks you to find reasons why derivative contracts based on HDD and CDD would be of interest to different businesses. These businesses likely have an interest to hedge their exposure to temperature fluctuations that are detrimental to their business. In answering the problem, it is useful to ask the question: what scenario hurts the company and how can it protect itself?

- 1. A soft drink manufacturer probably sells more drinks when it is abnormally hot, i.e., when the HDD/CDD is lower/higher than normal. She dislikes days that are abnormally cold (when the HDD is higher than normal) because people are likely to drink less, and her business suffers. Therefore, she will be interested in HDD futures contract that pays when the HDD is abnormally high (this, as you will see, will be a "long" futures position) because it will make payments when her usual business is slow.
- 2. A ski-resort operator may fear large losses if it is warmer than usual. It is detrimental to her business if it does not snow in the beginning of the season, or if the snow is melting too fast at the end of the season. She will be interested in a derivatives contract that will pay off when the weather is abnormally warm (when the HDD is low or the CDD is high). An example would be to take a position against the HDD (this will be a "short" futures position).
- 3. During the summer months, an electric utility company, such as one in the southern United States, will sell a lot of energy during days of excessive heat, because people will use their air conditioners, refrigerators and fans more often, thus consuming a lot of energy and increasing profits for the utility company. In this scenario, the utility company will have less business during relatively colder days, when the CDD will be relatively low. Hence, taking a position against the CDD (a short futures position) futures offers a possibility to hedge the company's risk.

Alternatively, we may think of a utility provider in the northeast United States during the winter months, in a region where people use many additional electric heaters. This utility provider will make more money during unusually cold days, and may be interested in going against the HDD contract (i.e., a short position in an HDD futures contract), because that contract pays off if the primary business suffers (when the HDD is low).

4. An amusement park operator fears bad weather and cold days, because people will abstain from going to the amusement park during cold days. She will short CDD futures (this will pay off when the CDD is relatively low) to offset her losses from ticket sales with gains from the futures contract.

# Question 1.4

A variety of counter-parties are imaginable. For one, we could think about speculators who have differences in opinion and who do not believe that we will have excessive temperature variations during the life of the futures contracts. Thus, they are willing to take the opposing side, receiving a payoff if the weather is stable. Even without different opinions or information, speculators might be willing to take on risk if the price is favorable enough (they are rewarded a risk premium for taking on the risk).

Alternatively, there may be opposing hedging needs. Compare the ski-resort operator and the soft-drink manufacturer. High values of HDD (relatively cold winter weather) will adversely affect the soft-drink manufacturer and, at the same time, positively affect the ski-resort operator's profits. Low values of HDD leads to the opposite effect (higher profits for the soft-drink manufacturer and lower profits for the ski-resort operator). Hence the two can share their risk by taking opposite sides of the future contract; the soft-drink manufacturer will buy (technically go long) the HDD and the ski resort operator will sell (go short) the HDD futures contract.

# Question 1.5

1. Remember that the terminology bid and ask is formulated from the market makers perspective. Therefore, the price at which you can buy is called the ask price. Furthermore, you will have to pay the commission to your broker for the transaction. You pay:

 $($41.05 \times 100) + $20 = $4,125.00$ 

2. Similarly, you can sell at the market maker's bid price. You will again have to pay a commission, and your broker will deduct the commission from the sales price of the shares. You receive:

$$(\$40.95 \times 100) - \$20 = \$4,075.00$$

3. By buying and selling immediately you incur two commissions 2×\$20 as well as the bid-ask spread (\$10) which leads to a \$50 round-trip transaction cost. We could also use the previous two answers to calculate your round-trip transaction costs:

$$4,125.00 - 4,075.00 = 50$$

## Question 1.6

In this problem, the brokerage fee is variable, and depends on the actual dollar amount of the sale/purchase of the shares. The concept of the transaction cost remains the same. If you buy the shares, the commission is added to the amount you owe, and if you sell the shares, the commission is deducted from the proceeds of the sale. Before deriving an exact amount, the 0.3% commission fee should imply approximately a  $.003 \times \$40 = \$12$  commission, implying our costs being lower by \$8 per transaction. Here are the exact amounts:

1. 
$$(\$41.05 \times 100) + (\$41.05 \times 100) \times 0.003 = \$4,117.315$$
  
= \$4,117.32

2.  $(\$40.95 \times 100) - (\$40.95 \times 100) \times 0.003 = \$4,082.715$ = \$4,082.72

3. \$4,117.32 - \$4,082.72 = \$34.60

The variable (or proportional) brokerage fee is advantageous to us. Our round-trip transaction fees are reduced by \$15.40.

#### Question 1.7

In answering this question it is important to remember that the market maker provides a service to the market. He stands ready to buy shares into his inventory and sell shares out of his inventory, thus providing immediacy to the market. He is remunerated for this service by earning the bid-ask spread. The market maker buys the security at a price of \$100, and he sells it at a price of \$100.10. If he buys 100 shares of the security and immediately sells them to another party, he is earns a spread of:

 $100 \times (\$100.12 - \$100) = 100 \times \$0.12 = \$12.00.$ 

#### Question 1.8

A short sale of XYZ entails borrowing shares of XYZ and then selling them, receiving cash. Therefore, initially, we will receive the proceeds from the sale of the asset, less the proportional commission charge:

 $300 \times (\$30.19) - 300 \times (\$30.19) \times 0.005 = \$9,057 \times 0.995$ = \$9,011.72

When we close out the position, we will again incur the commission charge, which is added to the purchasing cost:

$$300 \times (\$29.87) + 300 \times (\$29.87) \times 0.005 = \$8,961 \times 1.005$$
  
= \$9,005.81

Finally, we subtract the cost of covering the short position from our initial proceeds to receive total profits: 9,011.72 - 9,005.81 = 5.91. We can see that the commission charge that we have to pay twice significantly reduces the profits we can make.

#### Question 1.9

1. A short sale of JKI stock entails borrowing shares of JKI and selling them at the bid price. Therefore, initially, we will receive the proceeds from the sale of the asset at the bid (ignoring the commissions and interest). After 180 days, we cover the short position by buying the JKI stock at the ask, earning the following profit:

$$400 \times (\$25.12) - 400 \times (\$23.06) = \$10,048 - \$9,224$$
  
= \\$824

2. We have to pay the commission twice. The commission will reduce our profit to:

 $400 \times (\$25.12) - 400 \times (\$25.12) \times 0.003 - 400 \times (\$23.06) - 400 \times (\$23.06) \times 0.003$ =  $\$10,048 \times 0.997 - \$9,224 \times 1.003$ = \$10,017.856 - \$9251.672= \$766.184.

3. The proceeds from short sales, minus the commission charge, are \$10,017.856 (or \$10,048 if you ignore the commission charge). Since the 6-month interest rate is given, and the period of our short sale is exactly half a year, we can directly calculate the interest we could earn (and that we now lose) on a deposit of \$10,017.856:

 $10,017.856 \times (0.03) = 300.5357$ = 300.54

or, without taking into account the commission charge:

 $10,048 \times (0.03) = 301.44.$ 

## Question 1.10

We learned from the main text that short selling is equivalent to borrowing money, and that a short seller will often have to deposit the proceeds of the short sale with the lender as collateral.

A short seller is entitled to earn interest on his collateral, and the interest rate he earns is called the short rebate in the stock market. Usually, the short rebate is close to the prevailing market interest rate. Sometimes, though, a particular stock is scarce and difficult to borrow. In this case, the short rebate is substantially less than the current market interest rate, and an equity lender can earn a nice profit in the form of the difference between the current market interest rate and the short rebate.

By signing an agreement as mentioned in the problem, you give your brokerage firm the possibility to act as an equity lender, using the shares of your account. Brokers want you to sign such an agreement because they can make additional profits.

# Question 1.11

If we borrow an asset from a lender, we have the obligation to make any payments to the lender that she is entitled to as a stockholder. As the lender is entitled to the dividend on the day the stock goes ex-dividend, but does not receive it from the company, because we have sold her stock, we must provide the dividend. This payment is tax-deductible for us.

In a perfect capital market, we would expect that the stock price falls exactly by the amount of the dividend on the ex-date. Therefore, we should not care.

However, two complications may arise. First, we may have borrowed a large amount of shares, and the increased dividend forces us to pay more to the lender, and we may not have the additional required money.

On a more subtle note, an unexpected increase of the dividend is a strong signal that the company is doing exceedingly well, and empirically, we observe sharp price increases after such announcements. As we have a short position in the stock, we lose money as the stock price increases on this unexpected information. Therefore, an unexpected increase in the dividend is very bad for our short position and we are likely to care!

As of the time of this writing, December 10, 2007, the website http://nysedata.com makes available (in spreadsheet form) short interest for all listed securities. The following two tables present the highest volume securities and the highest short interest securities (respectively) for November 15, 2007.

| Symbol | Stock Description    | Avg Daily Vol | <b>Curr Short Pos</b> | Prev Short Pos |
|--------|----------------------|---------------|-----------------------|----------------|
| NT     | Nortel Networks Corp | 33,220,871    | 48,597,371            | 42,153,922     |
| LU     | Lucent Technologies  | 33,100,676    | 257,277,501           | 266,690,377    |
| GE     | General Electric Co  | 29,314,795    | 31,575,513            | 41,624,381     |
| MOT    | Motorola Inc         | 16,278,367    | 35,298,966            | 39,095,317     |
| PFE    | Pfizer Inc           | 15,619,357    | 42,773,927            | 40,830,684     |
| EMC    | EMC Corp             | 14,514,905    | 18,237,409            | 18,986,196     |
| TXN    | Texas Instruments    | 13,588,924    | 22,251,419            | 21,985,184     |
| TWX    | Time Warner Inc      | 12,226,790    | 74,589,776            | 63,767,969     |
| XOM    | Exxon Mobil Corp     | 11,642,757    | 32,804,323            | 30,111,721     |
| WMT    | Wal-Mart Stores Inc  | 11,395,419    | 23,102,572            | 27,884,228     |
| Symbol | Stock Description    | Avg Daily Vol | Curr Short Pos        | Prev Short Pos |
| LU     | Lucent Technologies  | 33,100,676    | 257,277,501           | 266,690,377    |
| CPN    | Calpine Corporation  | 8,462,043     | 125,006,771           | 133,510,647    |
| F      | Ford Motor Company   | 10,115,395    | 96,776,525            | 97,740,673     |
| TWX    | Time Warner Inc.     | 12,226,790    | 74,589,776            | 63,767,969     |
| AMD    | Advanced Micro Dev   | 9,818,933     | 64,113,540            | 68,281,617     |
| TYC    | Tyco Intl Ltd        | 7,893,905     | 58,547,462            | 57,628,171     |
| DAL    | Delta Air Lines, Inc | 3,650,762     | 58,547,111            | 56,675,178     |
| FON    | Sprint Corp          | 6,601,124     | 54,718,070            | 52,387,494     |
| AMR    | Amr Corp             | 4,147,314     | 50,670,382            | 45,949,723     |
| MU     | Micro Technology     | 8,760,576     | 50,494,618            | 43,690,088     |

A large change in short interest (as in Time Warner's increase of General Electric's decrease) might contain information (e.g., the market's optimism or pessimism), though reasons for such changes are very difficult, if not impossible, to determine. In general, stocks that are currently going through a period of volatility due to significant events (mergers and acquisition, dividend dates, industry dynamics, etc.) will have a particularly high short interest. It is theoretically possible to have short interest of more than 100%, because some market participants (e.g., market makers) have the ability to short sell a stock without having a locate, i.e., having someone who actually owns the stock and has agreed to lend it.

# Question 1.13

We are interested in borrowing the asset "money." Therefore, we go to an owner (or, if you prefer, to a collector) of the asset, called Bank. The Bank provides the \$100 of the asset money in digital form by increasing our bank account. We sell the digital money by going to the ATM and withdrawing cash. After 90 days, we buy back the digital money for \$102, by depositing cash into our bank account. The lender is repaid, and we have covered our short position.

We are interested in borrowing the asset "money" to buy a house. Therefore, we go to an owner of the asset, called Bank. The Bank provides the dollar amount, say \$250,000, in digital form in our mortgage account. As \$250,000 is a large amount of money, the bank is subject to substantial credit risk (e.g., we may lose our job) and demands collateral. Although the money itself is not subject to large variations in price (besides inflation risk, it is difficult to imagine a reason for money to vary in value), the Bank knows that we want to buy a house, and real estate prices vary substantially. Therefore, the Bank wants more collateral than the \$250,000 they are lending.

In fact, as the Bank is only lending up to 80% of the value of the house, we could get a mortgage of \$250,000 for a house that is worth  $$250,000 \div 0.8 = $312,500$ . We see that the bank factored in a haircut of \$312,500 - \$250,000 = \$62,500 to protect itself from credit risk and adverse fluctuations in property prices.

We buy back the asset money over a long horizon of time by reducing our mortgage through annuity payments.