CHAPTER 1

1.1 We will illustrate two different methods for solving this problem: (1) separation of variables, and (2) Laplace transform.

$$\frac{dv}{dt} = g - \frac{c}{m}v$$

Separation of variables: Separation of variables gives

$$\int \frac{1}{g - \frac{c}{m}v} dv = \int dt$$

The integrals can be evaluated as

$$-\frac{\ln\left(g - \frac{c}{m}v\right)}{c/m} = t + C$$

where C = a constant of integration, which can be evaluated by applying the initial condition to yield

$$C = -\frac{\ln\left(g - \frac{c}{m}v(0)\right)}{c/m}$$

which can be substituted back into the solution

$$-\frac{\ln\left(g - \frac{c}{m}v\right)}{c/m} = t - \frac{\ln\left(g - \frac{c}{m}v(0)\right)}{c/m}$$

This result can be rearranged algebraically to solve for v,

$$v = v(0)e^{-(c/m)t} + \frac{mg}{c}\left(1 - e^{-(c/m)t}\right)$$

where the first part is the general solution and the second part is the particular solution for the constant forcing function due to gravity. For the case where, $v(0) = 0$, the solution reduces to Eq. (1.10)

$$v = \frac{mg}{c}\left(1 - e^{-(c/m)t}\right)$$

Laplace transform solution: An alternative solution is provided by applying Laplace transform to the differential equation to give

$$sV(s) - v(0) = \frac{g}{s} - \frac{c}{m}V(s)$$

Solve algebraically for the transformed velocity

$$V(s) = \frac{v(0)}{s + c/m} + \frac{g}{s(s + c/m)} \qquad (1)$$

The second term on the right of the equal sign can be expanded with partial fractions

$$\frac{g}{s(s + c/m)} = \frac{A}{s} + \frac{B}{s + c/m} = \frac{A(s + c/m) + Bs}{s(s + c/m)} \qquad (2)$$

By equating like terms in the numerator, the following must hold

$$g = A\frac{c}{m} \qquad\qquad 0 = As + Bs$$

The first equation can be solved for $A = mg/c$. According to the second equation, $B = -A$, so $B = -mg/c$. Substituting these back into (2) gives

$$\frac{g}{s(s + c/m)} = \frac{mg/c}{s} - \frac{mg/c}{s + c/m}$$

This can be substituted into Eq. 1 to give

$$V(s) = \frac{v(0)}{s + c/m} + \frac{mg/c}{s} - \frac{mg/c}{s + c/m}$$

Taking inverse Laplace transforms yields

$$v(t) = v(0)e^{-(c/m)t} + \frac{mg}{c} + \frac{mg}{c}e^{-(c/m)t}$$

or collecting terms

$$v(t) = v(0)e^{-(c/m)t} + \frac{mg}{c}\left(1 - e^{-(c/m)t}\right)$$

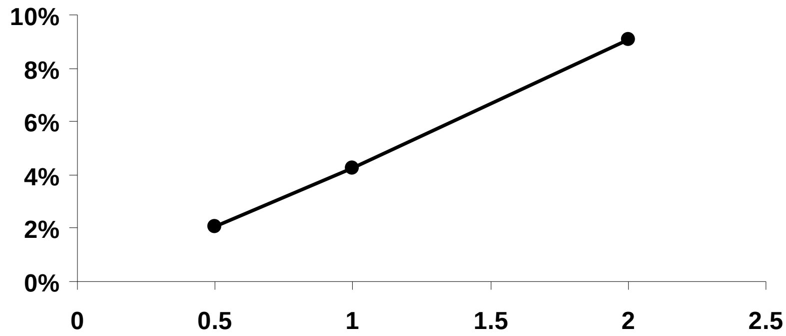
1.2 At $t = 8$ s, the analytical solution is 41.137 (Example 1.1). The relative error can be calculated with

$$\text{absolute relative error} = \left|\frac{\text{analytical} - \text{numerical}}{\text{analytical}}\right| \times 100\%$$

The numerical results are:

step	v(8)	absolute relative error
2	44.8700	9.074%
1	42.8931	4.268%
0.5	41.9901	2.073%

The error versus step size can then be plotted as



Thus, halving the step size approximately halves the error.

1.3 ***(a)*** You are given the following differential equation with the initial condition, $v(t = 0) = 0$,

$$\frac{dv}{dt} = g - \frac{c'}{m}v^2$$

Multiply both sides by m/c' gives

$$\frac{m}{c'}\frac{dv}{dt} = \frac{m}{c'}g - v^2$$

Define $a = \sqrt{mg / c'}$

$$\frac{m}{c'}\frac{dv}{dt} = a^2 - v^2$$

Integrate by separation of variables,

$$\int \frac{dv}{a^2 - v^2} = \int \frac{c'}{m}dt$$

A table of integrals can be consulted to find that

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{a}\tanh^{-1}\frac{x}{a}$$

Therefore, the integration yields

$$\frac{1}{a}\tanh^{-1}\frac{v}{a} = \frac{c'}{m}t + C$$

If $v = 0$ at $t = 0$, then because $\tanh^{-1}(0) = 0$, the constant of integration $C = 0$ and the solution is

$$\frac{1}{a}\tanh^{-1}\frac{v}{a} = \frac{c'}{m}t$$

This result can then be rearranged to yield

$$v = \sqrt{\frac{gm}{c'}}\tanh\left(\sqrt{\frac{gc'}{m}}t\right)$$

(b) Using Euler's method, the first two steps can be computed as

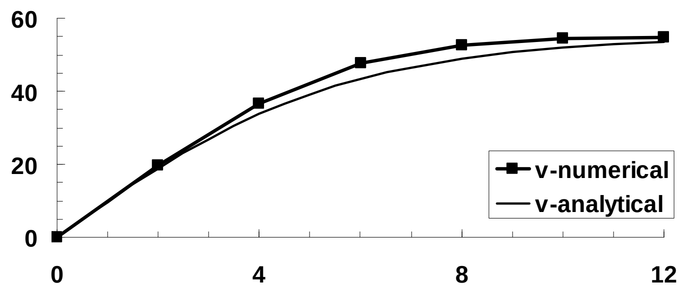
$$v(2) = 0 + \left[9.81 - \frac{0.22}{68.1}(0)^2\right]2 = 19.62$$

$$v(4) = 19.62 + \left[9.81 - \frac{0.22}{68.1}(19.62)^2\right]2 = 36.75284$$

The computation can be continued and the results summarized along with the analytical result as:

t	*v-numerical*	*dv/dt*	*v-analytical*
0	0	9.81	0
2	19.62	8.56642	18.83093
4	36.75284	5.446275	33.72377
6	47.64539	2.476398	43.46492
8	52.59819	0.872478	49.06977
10	54.34314	0.269633	52.05938
12	54.88241	0.079349	53.58978
∞	55.10572	0.022993	55.10572

A plot of the numerical and analytical results can be developed



1.4 $v(t) = \frac{gm}{c}(1 - e^{-(c/m)t})$

jumper #1: $v(t) = \frac{9.81(70)}{12}(1 - e^{-(12/70)9}) = 44.99204$

jumper #2: $44.99204 = \frac{9.81(80)}{15}(1 - e^{-(15/80)t})$

$$44.99204 = 52.32 - 52.32e^{-0.1875t}$$

$$0.14006 = e^{-0.1875t}$$

$$\ln 0.14006 = -0.1875t$$

$$t = \frac{\ln 0.14006}{-0.1875} = 10.4836 \text{ s}$$

1.5 Before the chute opens ($t < 10$), Euler's method can be implemented as

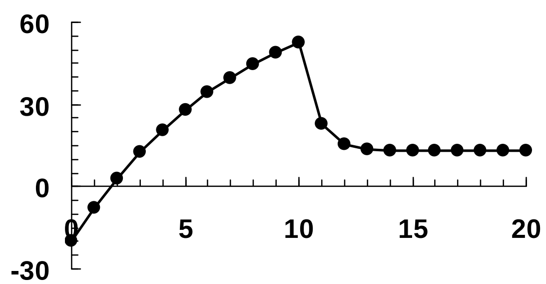
$$v(t+\Delta t) = v(t) + \left[9.81 - \frac{10}{80}v(t)\right]\Delta t$$

After the chute opens ($t \geq 10$), the drag coefficient is changed and the implementation becomes

$$v(t+\Delta t) = v(t) + \left[9.81 - \frac{60}{80}v(t)\right]\Delta t$$

Here is a summary of the results along with a plot:

	Chute closed			**Chute opened**	
t	***v***	***dv/dt***	***t***	***v***	***dv/dt***
0	-20.0000	12.3100	10	52.5723	-29.6192
1	-7.6900	10.7713	11	22.9531	-7.4048
2	3.0813	9.4248	12	15.5483	-1.8512
3	12.5061	8.2467	13	13.6971	-0.4628
4	20.7528	7.2159	14	13.2343	-0.1157
5	27.9687	6.3139	15	13.1186	-0.0289
6	34.2826	5.5247	16	13.0896	-0.0072
7	39.8073	4.8341	17	13.0824	-0.0018
8	44.6414	4.2298	18	13.0806	-0.0005
9	48.8712	3.7011	19	13.0802	-0.0001
			20	13.0800	0.0000



1.6 (*a*) This is a transient computation. For the period ending June 1:

Balance = Previous Balance + Deposits – Withdrawals + Interest
Balance = 1522.33 + 220.13 – 327.26 + 0.01(1522.33) = 1430.42

The balances for the remainder of the periods can be computed in a similar fashion as tabulated below:

Date	**Deposit**	**Withdrawal**	**Interest**	**Balance**
1-May				$1,522.33
	$220.13	$327.26	$15.22	
1-Jun				$1,430.42
	$216.80	$378.51	$14.30	
1-Jul				$1,283.02
	$450.35	$106.80	$12.83	
1-Aug				$1,639.40
	$127.31	$350.61	$16.39	
1-Sep				$1,432.49

(*b*) $\frac{dB}{dt} = D(t) - W(t) - iB$

(c) for $t = 0$ to 0.5:

$$\frac{dB}{dt} = 220.13 - 327.26 + 0.01(1522.33) = -91.91$$

$$B(0.5) = 1522.33 - 91.91(0.5) = 1476.38$$

for $t = 0.5$ to 1:

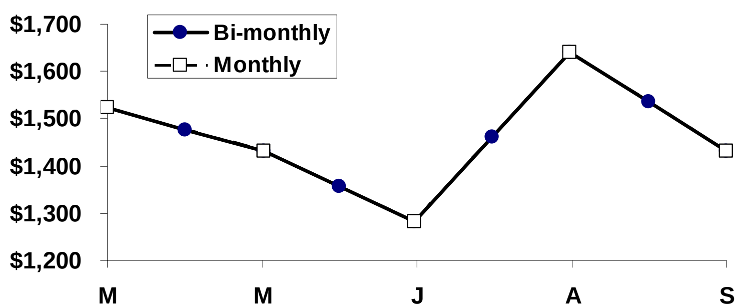
$$\frac{dB}{dt} = 220.13 - 327.260 + 0.01(1476.38) = -92.37$$

$$B(0.5) = 1476.38 - 92.37(0.5) = 1430.19$$

The balances for the remainder of the periods can be computed in a similar fashion as tabulated below:

Date	Deposit	Withdrawal	Interest	*dB/dt*	Balance
1-May	$220.13	$327.26	$15.22	-$91.91	$1,522.33
16-May	$220.13	$327.26	$14.76	-$92.37	$1,476.38
1-Jun	$216.80	$378.51	$14.30	-$147.41	$1,430.19
16-Jun	$216.80	$378.51	$13.56	-$148.15	$1,356.49
1-Jul	$450.35	$106.80	$12.82	$356.37	$1,282.42
16-Jul	$450.35	$106.80	$14.61	$358.16	$1,460.60
1-Aug	$127.31	$350.61	$16.40	-$206.90	$1,639.68
16-Aug	$127.31	$350.61	$15.36	-$207.94	$1,536.23
1-Sep					$1,432.26

(d) As in the plot below, the results of the two approaches are very close.



1.7 (a) The first two steps are

$$c(0.1) = 100 - 0.175(100)0.1 = 98.25 \text{ Bq/L}$$

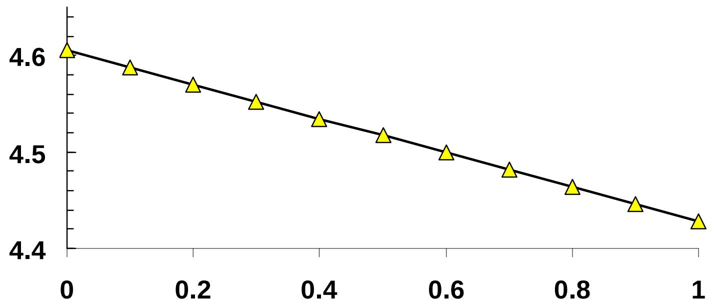
$$c(0.2) = 98.25 - 0.175(98.25)0.1 = 96.5306 \text{ Bq/L}$$

The process can be continued to yield

t	*c*	*dc/dt*
0	100.0000	-17.5000
0.1	98.2500	-17.1938
0.2	96.5306	-16.8929
0.3	94.8413	-16.5972
0.4	93.1816	-16.3068
0.5	91.5509	-16.0214
0.6	89.9488	-15.7410
0.7	88.3747	-15.4656

0.8	86.8281	-15.1949
0.9	85.3086	-14.9290
1	83.8157	-14.6678

(b) The results when plotted on a semi-log plot yields a straight line



The slope of this line can be estimated as

$$\frac{\ln(83.8157) - \ln(100)}{1} = -0.17655$$

Thus, the slope is approximately equal to the negative of the decay rate. If we had used a smaller step size, the result would be more exact.

1.8

$$Q_{\text{students}} = 35 \text{ ind} \times 80 \frac{\text{J}}{\text{ind s}} \times 20 \text{ min} \times 60 \frac{\text{s}}{\text{min}} \times \frac{\text{kJ}}{1000 \text{ J}} = 3{,}360 \text{ kJ}$$

$$m = \frac{PV\text{Mwt}}{RT} = \frac{(101.325 \text{ kPa})(11\text{m} \times 8\text{m} \times 3\text{m} - 35 \times 0.075 \text{ m}^3)(28.97 \text{ kg/kmol})}{(8.314 \text{ kPa m}^3 / (\text{kmol K})((20 + 273.15)\text{K})} = 314.796 \text{ kg}$$

$$\Delta T = \frac{Q_{\text{students}}}{mC_v} = \frac{3{,}360 \text{ kJ}}{(314.796 \text{ kg})(0.718 \text{ kJ/(kg K)})} = 14.86571 \text{ K}$$

Therefore, the final temperature is 20 + 14.86571 = 34.86571°C**.**

1.9 The first two steps yield

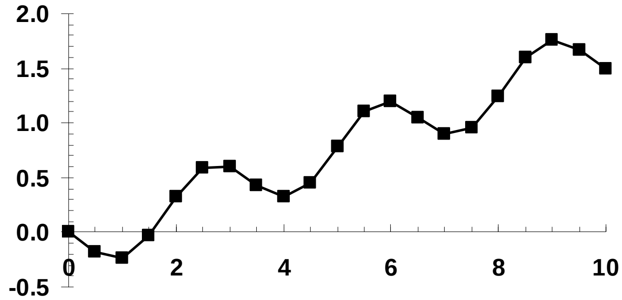
$$y(0.5) = 0 + \left[3\frac{450}{1250}\sin^2(0) - \frac{450}{1250}\right]0.5 = \ 0 + (-0.36)\ 0.5 = -0.18$$

$$y(1) = -0.18 + \left[3\frac{450}{1250}\sin^2(0.5) - \frac{450}{1250}\right]0.5 = \ -0.18 + (-0.11176)\ 0.5 = -0.23588$$

The process can be continued to give the following table and plot:

t	*y*	*dy/dt*	*t*	*y*	*dy/dt*
0	0.00000	-0.36000	5.5	1.10271	0.17761
0.5	-0.18000	-0.11176	6	1.19152	-0.27568
1	-0.23588	0.40472	6.5	1.05368	-0.31002
1.5	-0.03352	0.71460	7	0.89866	0.10616
2	0.32378	0.53297	7.5	0.95175	0.59023
2.5	0.59026	0.02682	8	1.24686	0.69714
3	0.60367	-0.33849	8.5	1.59543	0.32859

3.5	0.43443	-0.22711	9	1.75972	-0.17657
4	0.32087	0.25857	9.5	1.67144	-0.35390
4.5	0.45016	0.67201	10	1.49449	-0.04036
5	0.78616	0.63310			



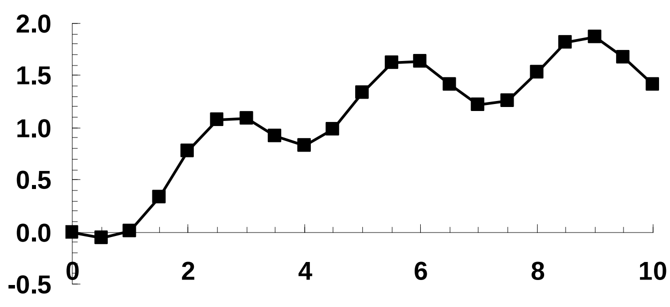
1.10 The first two steps yield

$$y(0.5) = 0 + \left[3\frac{450}{1250}\sin^2(0) - \frac{150(1+0)^{1.5}}{1250} \right] 0.5 = 0 - 0.12(0.5) = \ -0.06$$

$$y(1) = -0.06 + \left[3\frac{450}{1250}\sin^2(0.5) - \frac{150(1-0.06)^{1.5}}{1250} \right] 0.5 = -0.06 + 0.13887(0.5) = 0.00944$$

The process can be continued to give

t	*y*	*dy/dt*	*t*	*y*	*dy/dt*
0	0.00000	-0.12000	5.5	1.61981	0.02876
0.5	-0.06000	0.13887	6	1.63419	-0.42872
1	0.00944	0.64302	6.5	1.41983	-0.40173
1.5	0.33094	0.89034	7	1.21897	0.06951
2	0.77611	0.60892	7.5	1.25372	0.54423
2.5	1.08058	0.02669	8	1.52584	0.57542
3	1.09392	-0.34209	8.5	1.81355	0.12227
3.5	0.92288	-0.18708	9	1.87468	-0.40145
4	0.82934	0.32166	9.5	1.67396	-0.51860
4.5	0.99017	0.69510	10	1.41465	-0.13062
5	1.33772	0.56419			



1.11 When the water level is above the outlet pipe, the volume balance can be written as

$$\frac{dV}{dt} = 3\sin^2(t) - 3(y - y_{\text{out}})^{1.5}$$

In order to solve this equation, we must relate the volume to the level. To do this, we recognize that the volume of a cone is given by $V = \pi r^2 y/3$. Defining the side slope as $s = y_{top}/r_{top}$, the radius can be related to the level ($r = y/s$) and the volume can be reexpressed as

$$V = \frac{\pi}{3s^2} y^3$$

which can be solved for

$$y = \sqrt[3]{\frac{3s^2 V}{\pi}} \tag{1}$$

and substituted into the volume balance

$$\frac{dV}{dt} = 3\sin^2(t) - 3\left(\sqrt[3]{\frac{3s^2 V}{\pi}} - y_{\text{out}}\right)^{1.5} \tag{2}$$

For the case where the level is below the outlet pipe, outflow is zero and the volume balance simplifies to

$$\frac{dV}{dt} = 3\sin^2(t) \tag{3}$$

These equations can then be used to solve the problem. Using the side slope of $s = 4/2.5 = 1.6$, the initial volume can be computed as

$$V(0) = \frac{\pi}{3(1.6)^2} 0.8^3 = 0.20944 \text{ m}^3$$

For the first step, $y < y_{\text{out}}$ and Eq. (3) gives

$$\frac{dV}{dt}(0) = 3\sin^2(0) = 0$$

and Euler's method yields

$$V(0.5) = V(0) + \frac{dV}{dt}(0)\Delta t = 0.20944 + 0(0.5) = 0.20944$$

For the second step, Eq. (3) still holds and

$$\frac{dV}{dt}(0.5) = 3\sin^2(0.5) = 0.689547$$

$$V(1) = V(0.5) + \frac{dV}{dt}(0.5)\Delta t = 0.20944 + 0.689547(0.5) = 0.554213$$

Equation (1) can then be used to compute the new level,

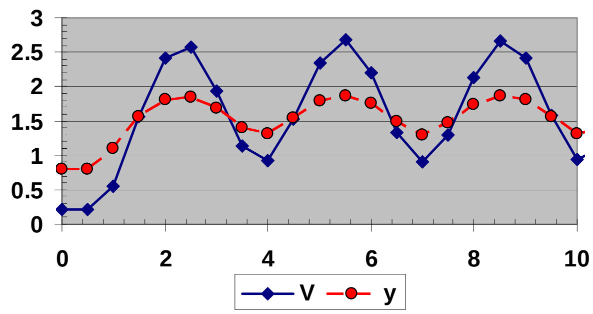
$$y = \sqrt[3]{\frac{3(1.6)^2(0.554213)}{\pi}} = 1.106529 \text{ m}$$

Because this level is now higher than the outlet pipe, Eq. (2) holds for the next step

$$\frac{dV}{dt}(1) = 2.12422 - 3(1.106529 - 1)^{1.5} = 2.019912$$
$$V(1.5) = 0.554213 + 2.019912(0.5) = 1.564169$$

The remainder of the calculation is summarized in the following table and figure.

t	***Q*in**	***V***	***y***	***Q*out**	***dV/dt***
0	0	0.20944	0.8	0	0
0.5	0.689547	0.20944	0.8	0	0.689547
1	2.12422	0.554213	1.106529	0.104309	2.019912
1.5	2.984989	1.564169	1.563742	1.269817	1.715171
2	2.480465	2.421754	1.809036	2.183096	0.29737
2.5	1.074507	2.570439	1.845325	2.331615	-1.25711
3	0.059745	1.941885	1.680654	1.684654	-1.62491
3.5	0.369147	1.12943	1.40289	0.767186	-0.39804
4	1.71825	0.93041	1.31511	0.530657	1.187593
4.5	2.866695	1.524207	1.55031	1.224706	1.641989
5	2.758607	2.345202	1.78977	2.105581	0.653026
5.5	1.493361	2.671715	1.869249	2.431294	-0.93793
6	0.234219	2.202748	1.752772	1.95937	-1.72515
6.5	0.13883	1.340173	1.48522	1.013979	-0.87515
7	1.294894	0.902598	1.301873	0.497574	0.79732
7.5	2.639532	1.301258	1.470703	0.968817	1.670715
8	2.936489	2.136616	1.735052	1.890596	1.045893
8.5	1.912745	2.659563	1.866411	2.419396	-0.50665
9	0.509525	2.406237	1.805164	2.167442	-1.65792
9.5	0.016943	1.577279	1.568098	1.284566	-1.26762
10	0.887877	0.943467	1.321233	0.5462	0.341677



1.12 (a) The force balance can be written as:

$$m\frac{dv}{dt} = -mg(0)\frac{R^2}{(R+x)^2} + c_d v|v|$$

Dividing by mass gives

$$\frac{dv}{dt} = -g(0)\frac{R^2}{(R+x)^2} + \frac{c_d}{m} v|v|$$

(b) Recognizing that $dx/dt = v$, the chain rule is

$$\frac{dv}{dt} = v\frac{dv}{dx}$$

Setting drag to zero and substituting this relationship into the force balance gives

$$\frac{dv}{dx} = -\frac{g(0)}{v}\frac{R^2}{(R+x)^2}$$

(c) Using separation of variables

$$v\,dv = -g(0)\frac{R^2}{(R+x)^2}dx$$

Integrating gives

$$\frac{v^2}{2} = g(0)\frac{R^2}{R+x} + C$$

Applying the initial condition yields

$$\frac{v_0^2}{2} = g(0)\frac{R^2}{R+0} + C$$

which can be solved for $C = v_0^2/2 - g(0)R$, which can be substituted back into the solution to give

$$\frac{v^2}{2} = g(0)\frac{R^2}{R+x} + \frac{v_0^2}{2} - g(0)R$$

or

$$v = \pm\sqrt{v_0^2 + 2g(0)\frac{R^2}{R+x} - 2g(0)R}$$

Note that the plus sign holds when the object is moving upwards and the minus sign holds when it is falling.

(d) Euler's method can be developed as

$$v(x_{i+1}) = v(x_i) + \left[-\frac{g(0)}{v(x_i)}\frac{R^2}{(R+x_i)^2}\right](x_{i+1} - x_i)$$

The first step can be computed as

$$v(10,000) = 1,500 + \left[-\frac{9.81}{1,500}\frac{(6.37\times10^6)^2}{(6.37\times10^6+0)^2}\right](10,000-0) = 1,500 + (-0.00654)10,000 = 1434.600$$

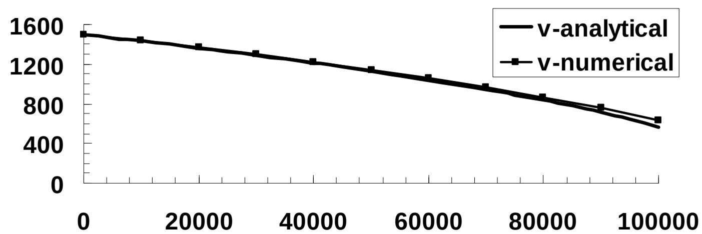
The remainder of the calculations can be implemented in a similar fashion as in the following table

x	*v*	*dv/dx*	*v*-analytical
0	1500.000	-0.00654	1500.000
10000	1434.600	-0.00682	1433.216
20000	1366.433	-0.00713	1363.388
30000	1295.089	-0.00750	1290.023
40000	1220.049	-0.00794	1212.475
50000	1140.643	-0.00847	1129.884
60000	1055.973	-0.00912	1041.049
70000	964.798	-0.00995	944.206
80000	865.317	-0.01106	836.579
90000	754.742	-0.01264	713.299
100000	628.359	-0.01513	564.197

For the analytical solution, the value at 10,000 m can be computed as

$$v = \sqrt{1{,}500^2 + 2(9.81)\frac{(6.37\times10^6)^2}{(6.37\times10^6 + 10{,}000)} - 2(9.81)(6.37\times10^6)} = 1433.216$$

The remainder of the analytical values can be implemented in a similar fashion as in the last column of the above table. The numerical and analytical solutions can be displayed graphically.



1.13 The volume of the droplet is related to the radius as

$$V = \frac{4\pi r^3}{3} \tag{1}$$

This equation can be solved for radius as

$$r = \sqrt[3]{\frac{3V}{4\pi}} \tag{2}$$

The surface area is

$$A = 4\pi r^2 \tag{3}$$

Equation (2) can be substituted into Eq. (3) to express area as a function of volume

$$A = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3}$$

This result can then be substituted into the original differential equation,

$$\frac{dV}{dt} = -k4\pi\left(\frac{3V}{4\pi}\right)^{2/3} \tag{4}$$

The initial volume can be computed with Eq. (1),

$$V = \frac{4\pi r^3}{3} = \frac{4\pi(2.5)^3}{3} = 65.44985 \text{ mm}^3$$

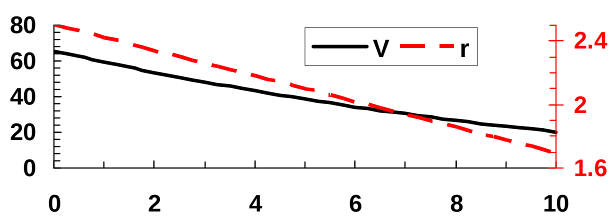
Euler's method can be used to integrate Eq. (4). For the first step, the result is

$$V(0.25) = V(0) + \frac{dV}{dt}(0) \times \Delta t = 65.44985 - 0.08(4)\pi\left(\frac{3(65.44985)}{4\pi}\right)^{2/3} \times 0.25$$
$$= 65.44985 - 6.28319(0.25) = 63.87905$$

Here are the beginning and ending steps

t	*V*	*dV/dt*
0	65.44985	-6.28319
0.25	63.87905	-6.18225
0.5	62.33349	-6.08212
0.75	60.81296	-5.98281
1	59.31726	-5.8843
•		
•		
•		
9	23.35079	-3.16064
9.25	22.56063	-3.08893
9.5	21.7884	-3.01804
9.75	21.03389	-2.94795
10	20.2969	-2.87868

A plot of the results is shown below. We have included the radius on this plot (dashed line and right scale):



Eq. (2) can be used to compute the final radius as

$$r = \sqrt[3]{\frac{3(20.2969)}{4\pi}} = 1.692182$$

Therefore, the average evaporation rate can be computed as

$$k = \frac{(2.5 - 1.692182) \text{ mm}}{10 \text{ min}} = 0.080782 \frac{\text{mm}}{\text{min}}$$

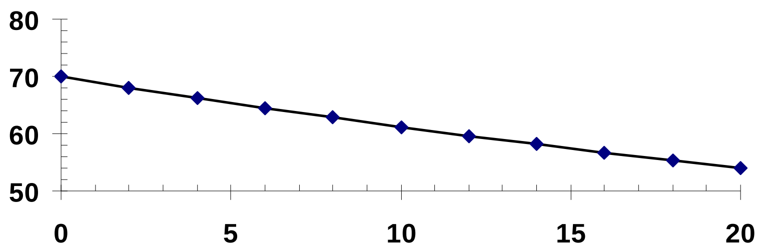
which is approximately equal to the given evaporation rate of 0.08 mm/min.

1.14 The first two steps can be computed as

$$T(1) = 70 + \left[-0.019(70-20)\right] 2 = 68 + (-0.95)2 = 68.1$$
$$T(2) = 68.1 + \left[-0.019(68.1-20)\right] 2 = 68.1 + (-0.9139)2 = 66.2722$$

The remaining results are displayed below along with a plot of the results.

t	*T*	*dT/dt*	*t*	*T*	*dT/dt*
0	70.00000	-0.95000	12.00000	59.62967	-0.75296
2	68.10000	-0.91390	14.00000	58.12374	-0.72435
4	66.27220	-0.87917	16.00000	56.67504	-0.69683
6	64.51386	-0.84576	18.00000	55.28139	-0.67035
8	62.82233	-0.81362	20.00000	53.94069	-0.64487
10	61.19508	-0.78271			



1.15 The pair of differential equations to be solved are

$$\frac{di}{dt} = -\frac{R}{L}i - \frac{1}{CL}q$$
$$\frac{dq}{dt} = i$$

or substituting the parameters

$$\frac{di}{dt} = -40i - 2{,}000q$$
$$\frac{dq}{dt} = i$$

The first step can be implemented by first using the differential equations to compute the slopes

$$\frac{di}{dt} = -40(0) - 2{,}000(1) = -2{,}000$$
$$\frac{dq}{dt} = 0$$

Then, Euler's method can be applied as

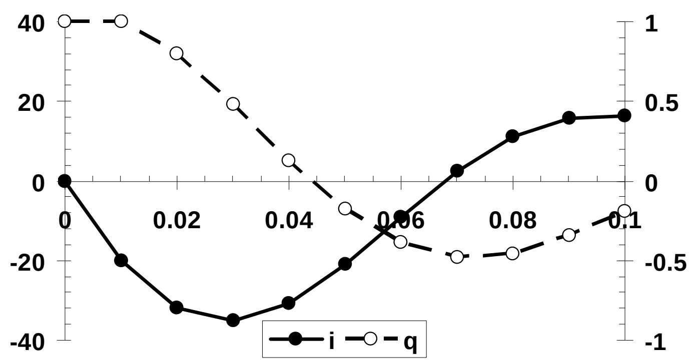
$$i(0.01) = 0 - 2{,}000(0.01) = -20$$
$$q(0.01) = 1 + 0(0.01) = 1$$

For the second step

$$\frac{di}{dt} = -40(-20) - 2{,}000(1) = -1{,}200$$
$$\frac{dq}{dt} = -20$$
$$i(0.02) = -20 - 1{,}200(0.01) = -32$$
$$q(0.02) = 1 - 20(0.01) = 0.8$$

The remaining steps are summarized in the following table and plot:

t	*i*	*q*	*di/dt*	*dq/dt*
0	0	1	-2000	0
0.01	-20	1	-1200	-20
0.02	-32	0.8	-320	-32
0.03	-35.2	0.48	448	-35.2
0.04	-30.72	0.128	972.8	-30.72
0.05	-20.992	-0.1792	1198.08	-20.992
0.06	-9.0112	-0.38912	1138.688	-9.0112
0.07	2.37568	-0.47923	863.4368	2.37568
0.08	11.01005	-0.45548	470.5485	11.01005
0.09	15.71553	-0.34537	62.12813	15.71553
0.1	16.33681	-0.18822	-277.034	16.33681



1.16 (*a*) The solution of the differential equation is

$$N = N_0 e^{\mu t}$$

The doubling time can be computed as the time when $N = 2N_0$,

$$2N_0 = N_0 e^{\mu(20)}$$

$$\mu = \frac{\ln 2}{20 \text{ hrs}} = \frac{0.693}{20 \text{ hrs}} = 0.034657/\text{hr}$$

(*b*) The volume of an individual spherical cell is

$$\text{cell volume} = \frac{\pi d^3}{6} \qquad (1)$$

The total volume is

$$\text{volume} = \frac{\pi d^3}{6} N \qquad (2)$$

The rate of change of N is defined as

$$\frac{dN}{dt} = \mu N \qquad (3)$$

If $N = N_0$ at $t = 0$, Eq. 3 can be integrated to give

$$N = N_0 e^{\mu t} \qquad (4)$$

Therefore, substituting (4) into (2) gives an equation for volume

$$\text{volume} = \frac{\pi d^3}{6} N_0 e^{\mu t} \qquad (5)$$

(c) This equation can be solved for time

$$t = \frac{\ln \dfrac{6 \times \text{volume}}{\pi d^3 N_0}}{\mu} \qquad (6)$$

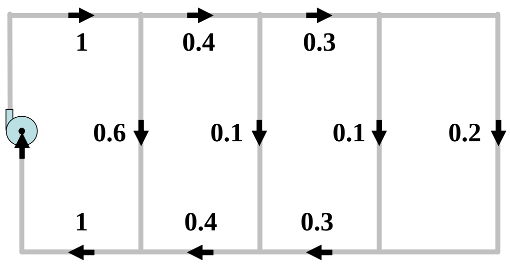
The volume of a 500 μm diameter tumor can be computed with Eq. 2 as 65,449,847. Substituting this value along with d = 20 μm, N_0 = 1 and μ = 0.034657/hr gives

$$t = \frac{\ln \left(\dfrac{6 \times 65{,}449{,}847}{\pi 20^3 (1)} \right)}{0.034657} = 278.63 \text{ hr} = 11.6 \text{ d} \qquad (6)$$

1.17 Continuity at the nodes can be used to determine the flows as follows:

$$Q_1 = Q_2 + Q_3 = 0.6 + 0.4 = 1.0 \ \text{m}^3/\text{s}$$
$$Q_{10} = Q_1 = 1.0 \ \text{m}^3/\text{s}$$
$$Q_9 = Q_{10} - Q_2 = 1.0 - 0.6 = 0.4 \ \text{m}^3/\text{s}$$
$$Q_4 = Q_9 - Q_8 = 0.4 - 0.3 = 0.1 \ \text{m}^3/\text{s}$$
$$Q_5 = Q_3 - Q_4 = 0.4 - 0.1 = 0.3 \ \text{m}^3/\text{s}$$
$$Q_6 = Q_5 - Q_7 = 0.3 - 0.2 = 0.1 \ \text{m}^3/\text{s}$$

Therefore, the final results are



1.18 (a) Substituting Eq. (1.10) into Eq. (P1.18) gives

$$\frac{dx}{dt} = \frac{gm}{c}(1 - e^{-(c/m)t})$$

Separation of variables gives

$$\int_0^x dx = \frac{gm}{c}\int_0^t 1 - e^{-(c/m)t}\,dt$$

Integration yields

$$x = \frac{gm}{c}t - \frac{gm^2}{c^2}(1 - e^{-(c/m)t})$$

(b) Euler's method can be applied for the first step as

$$\frac{dv}{dt}(0) = g - \frac{c}{m}v = 9.81 - \frac{12.5}{68.1}0 = 9.81$$
$$\frac{dx}{dt}(0) = v = 0$$
$$v(2) = v(0) + \frac{dv}{dt}(0)\Delta t = 0 + 9.81(2) = 19.62$$
$$x(2) = x(0) + \frac{dx}{dt}(0)\Delta t = 0 + 0(2) = 0$$

For the second step:

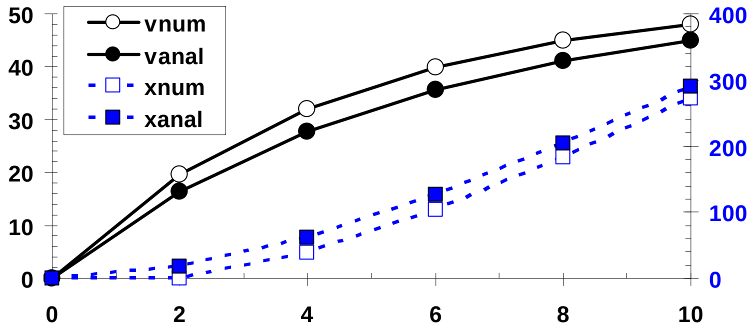
$$\frac{dv}{dt}(2) = 9.81 - \frac{12.5}{68.1}19.62 = 6.2087$$
$$\frac{dx}{dt}(0) = 19.62$$
$$v(4) = 19.62 + 6.2087(2) = 32.0374$$
$$x(4) = 0 + 19.62(2) = 39.24$$

The remaining steps can be computed in a similar fashion as tabulated below along with the analytical solution:

t	*vnum*	*xnum*	*dv/dt*	*dx/dt*	*vanal*	*xanal*
0	0.0000	0.0000	9.8100	0.0000	0.0000	0.0000
2	19.6200	0.0000	6.2087	19.6200	16.4217	17.4242

4	32.0374	39.2400	3.9294	32.0374	27.7976	62.3380
6	39.8962	103.3147	2.4869	39.8962	35.6781	126.2949
8	44.8700	183.1071	1.5739	44.8700	41.1372	203.4435
10	48.0179	272.8472	0.9961	48.0179	44.9189	289.7305

(*c*)



1.19 (a) For the constant temperature case, Newton's law of cooling is written as

$$\frac{dT}{dt} = -0.12(T-10)$$

The first two steps of Euler's methods are

$$T(0.5) = T(0) - \frac{dT}{dt}(0) \times \Delta t = 37 + 0.12(10-37)(0.5) = 37 - 3.2400 \times 0.50 = 35.3800$$
$$T(1) = 35.3800 + 0.12(10 - 35.3800)(0.5) = 35.3800 - 3.0456 \times 0.50 = 33.8572$$

The remaining calculations are summarized in the following table:

t	T_a	T	dT/dt
0:00	10	37.0000	-3.2400
0:30	10	35.3800	-3.0456
1:00	10	33.8572	-2.8629
1:30	10	32.4258	-2.6911
2:00	10	31.0802	-2.5296
2:30	10	29.8154	-2.3778
3:00	10	28.6265	-2.2352
3:30	10	27.5089	-2.1011
4:00	10	26.4584	-1.9750
4:30	10	25.4709	-1.8565
5:00	10	24.5426	-1.7451

(b) For this case, the room temperature can be represented as

$$T_a = 20 - 2t$$

where t = time (hrs). Newton's law of cooling is written as

$$\frac{dT}{dt} = -0.12(T - 20 + 2t)$$

The first two steps of Euler's methods are

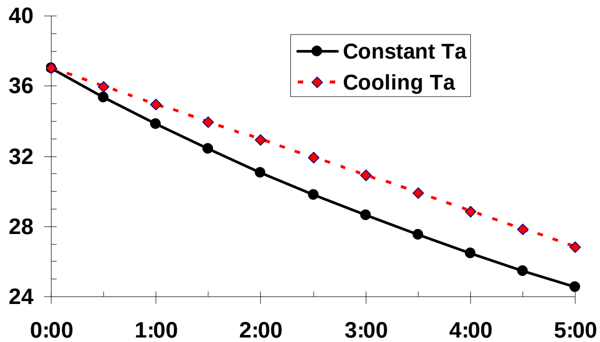
$$T(0.5) = 37 + 0.12(20 - 37)(0.5) = 37 - 2.040 \times 0.50 = 35.9800$$
$$T(1) = 35.9800 + 0.12(19 - 35.9800)(0.5) = 35.9800 - 2.0376 \times 0.50 = 34.9612$$

The remaining calculations are summarized in the following table:

t	*T_a*	*T*	*dT/dt*
0:00	20	37.0000	-2.0400
0:30	19	35.9800	-2.0376
1:00	18	34.9612	-2.0353
1:30	17	33.9435	-2.0332
2:00	16	32.9269	-2.0312
2:30	15	31.9113	-2.0294
3:00	14	30.8966	-2.0276
3:30	13	29.8828	-2.0259
4:00	12	28.8699	-2.0244
4:30	11	27.8577	-2.0229
5:00	10	26.8462	-2.0215

Comparison with (a) indicates that the effect of the room air temperature has a significant effect on the expected temperature at the end of the 5-hr period (difference = 26.8462 – 24.5426 = 2.3036°C).

(c) The solutions for (a) Constant T_a, and (b) Cooling T_a are plotted below:



1.20 (a)

$$\frac{dx}{dt} = v_x \quad \frac{dy}{dt} = v_y \qquad \frac{dv_x}{dt} = -\frac{c}{m} v_x \qquad \frac{dv_y}{dt} = g - \frac{c}{m} v_y$$

(b) The first step,

$$x(1) = x(0) + \frac{dx}{dt}\Delta t = 0 + 180(1) = 180$$
$$y(1) = y(0) + \frac{dy}{dt}\Delta t = -100 + 0(1) = -100$$
$$v_x(1) = v_x(0) + \frac{dv_x}{dt}\Delta t = 180 - \frac{12.5}{70}180(1) = 147.8571$$
$$v_y(1) = v_y(0) + \frac{dv_y}{dt}\Delta t = 0 + \left[9.81 - \frac{12.5}{70}(0)\right](1) = 9.81$$

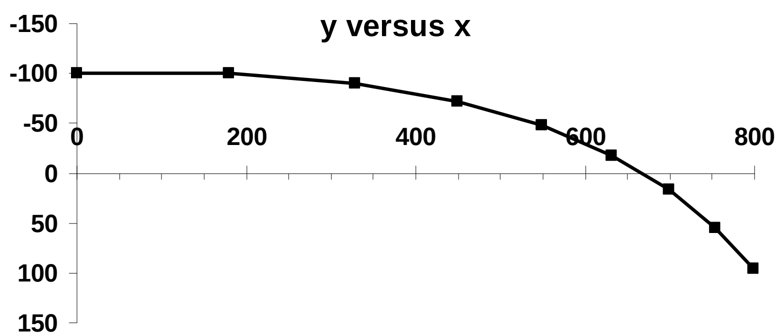
The second step

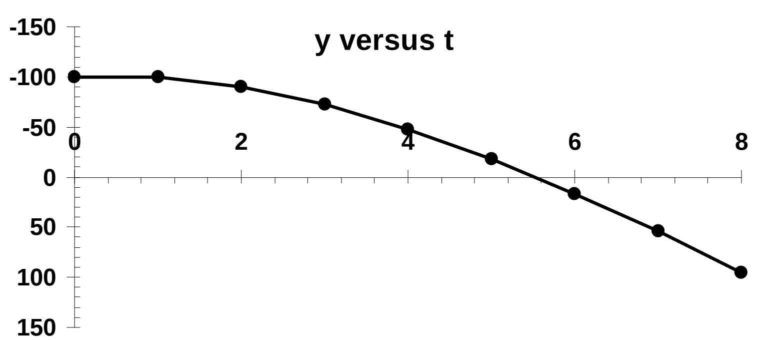
$$x(2)=180+147.8571(1)=327.8571$$
$$y(1)=-100+9.81(1)=-90.19$$
$$v_x(1)=147.8571-\frac{12.5}{70}147.8571(1)=121.4541$$
$$v_y(1)=9.81-\left[9.81-\frac{12.5}{70}(9.81)\right](1)=17.8682$$

These along with the remaining results can be tabulated as

t	*x*	*y*	v_x	v_y	*dx/dt*	*dy/dt*	dv_x/dt	dv_y/dt
0	0.0000	-100.0000	180.0000	0.0000	180.0000	0.0000	-32.1429	9.8100
1	180.0000	-100.0000	147.8571	9.8100	147.8571	9.8100	-26.4031	8.0582
2	327.8571	-90.1900	121.4541	17.8682	121.4541	17.8682	-21.6882	6.6192
3	449.3112	-72.3218	99.7659	24.4875	99.7659	24.4875	-17.8153	5.4372
4	549.0771	-47.8343	81.9505	29.9247	81.9505	29.9247	-14.6340	4.4663
5	631.0276	-17.9096	67.3165	34.3910	67.3165	34.3910	-12.0208	3.6687
6	698.3441	16.4814	55.2957	38.0598	55.2957	38.0598	-9.8742	3.0136
7	753.6398	54.5411	45.4215	41.0734	45.4215	41.0734	-8.1110	2.4755
8	799.0613	95.6145	37.3105	43.5488	37.3105	43.5488	-6.6626	2.0334
9	836.3718	139.1633	30.6479	45.5823	30.6479	45.5823	-5.4728	1.6703
10	867.0197	184.7456	25.1751	47.2526	25.1751	47.2526	-4.4955	1.3720

(c) The following plot indicates that the jumper will hit the ground in about *t* = 5.6 s at about *x* = 670 m.





1.21 (a) The force balance can be written as

$$m\frac{dv}{dt}=mg-\frac{1}{2}\rho v|v|AC_d$$

Dividing by mass gives

$$\frac{dv}{dt} = g - \frac{\rho A C_d}{2m} v|v| \tag{1}$$

The mass of the sphere is $\rho_s V$ where V = volume (m^3). The area and volume of a sphere are $\pi d^2/4$ and $\pi d^3/6$, respectively. Substituting these relationships gives

$$\frac{dv}{dt} = g - \frac{3\rho C_d}{4d\rho_s} v|v|$$

$$\frac{dx}{dt} = v$$

(b) The first step for Euler's method is

$$\frac{dv}{dt} = 9.81 - \frac{3(1.3)0.47}{4(1.2)2700}(-40)|-40| = 10.0363$$

$$\frac{dx}{dt} = -40$$

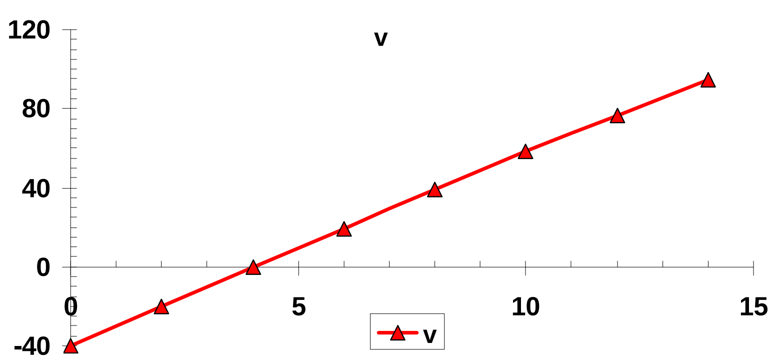
$$v = -40 + 10.0363(2) = -19.9274$$

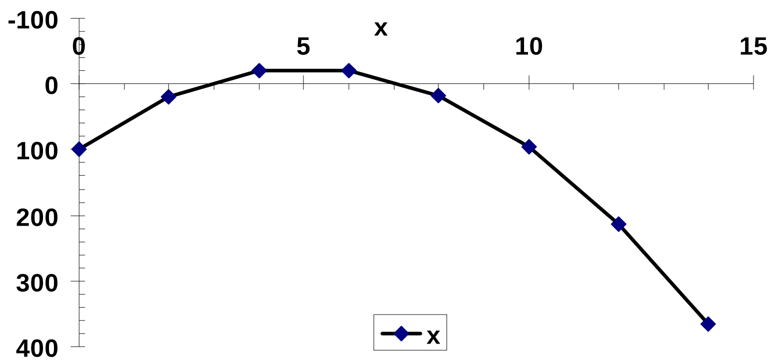
$$\frac{dx}{dt} = 100 - 40(2) = 20$$

The remaining steps are shown in the following table:

t	*x*	*v*	*dx/dt*	*dv/dt*
0	100.0000	-40.0000	-40.0000	10.0363
2	20.0000	-19.9274	-19.9274	9.8662
4	-19.8548	-0.1951	-0.1951	9.8100
6	-20.2450	19.4249	19.4249	9.7566
8	18.6049	38.9382	38.9382	9.5956
10	96.4813	58.1293	58.1293	9.3321
12	212.7399	76.7935	76.7935	8.9759
14	366.3269	94.7453	94.7453	8.5404

(c) The results can be graphed as (notice that we have reversed the axis for the distance, *x*, so that the negative elevations are upwards.





(d) Inspecting the differential equation for velocity (Eq. 1) indicates that the bulk drag coefficient is

$$c' = \frac{\rho A C_d}{2}$$

Therefore, for this case, because $A = \pi(1.2)^2/4 = 1.131$ m^2, the bulk drag coefficient is

$$c' = \frac{1.3(1.131)0.47}{2} = 0.3455\,\frac{\text{kg}}{\text{m}}$$

1.22 (a) A force balance on a sphere can be written as:

$$m\frac{dv}{dt} = F_{\text{gravity}} - F_{\text{buoyancy}} - F_{\text{drag}}$$

where

$$F_{\text{gravity}} = mg \qquad F_{\text{buoyancy}} = \rho V g \qquad F_{\text{drag}} = 3\pi\mu d v$$

Substituting the individual terms into the force balance yields

$$m\frac{dv}{dt} = mg - \rho V g - 3\pi\mu d v$$

Divide by *m*

$$\frac{dv}{dt} = g - \frac{\rho V g}{m} - \frac{3\pi\mu d v}{m}$$

Note that $m = \rho_s V$, so

$$\frac{dv}{dt} = g - \frac{\rho g}{\rho_s} - \frac{3\pi\mu d v}{\rho_s V}$$

The volume can be represented in terms of more fundamental quantities as $V = \pi d^3/6$. Substituting this relationship into the differential equation gives the final differential equation

$$\boxed{\frac{dv}{dt} = g\left(1 - \frac{\rho}{\rho_s}\right) - \frac{18\mu}{\rho_s d^2} v}$$

(b) At steady-state, the equation is

$$0 = g\left(1 - \frac{\rho}{\rho_s}\right) - \frac{18\mu}{\rho_s d^2} v$$

which can be solved for the terminal velocity

$$\boxed{v_\infty = \frac{g}{18}\frac{\rho_s - \rho}{\mu} d^2}$$

This equation is sometimes called *Stokes Settling Law*.

(c) Before computing the result, it is important to convert all the parameters into consistent units. For the present problem, the necessary conversions are

$$d = 10\ \mu\text{m} \times \frac{\text{m}}{10^6\ \mu\text{m}} = 10^{-5}\,\text{m} \qquad \rho = 1\frac{\text{g}}{\text{cm}^3} \times \frac{10^6\ \text{cm}^3}{\text{m}^3} \times \frac{\text{g}}{10^3\ \text{kg}} = 1000\frac{\text{kg}}{\text{m}^3}$$

$$\rho_s = 2.65\frac{\text{g}}{\text{cm}^3} \times \frac{10^6\ \text{cm}^3}{\text{m}^3} \times \frac{\text{g}}{10^3\ \text{kg}} = 2650\frac{\text{kg}}{\text{m}^3} \qquad \mu = 0.014\frac{\text{g}}{\text{cm s}} \times \frac{100\ \text{cm}}{\text{m}} \times \frac{\text{kg}}{1000\ \text{g}} = 0.0014\frac{\text{kg}}{\text{m s}}$$

The terminal velocity can then computed as

$$v_\infty = \frac{9.81}{18}\frac{2650 - 1000}{0.0014}(1\times 10^{-5})^2 = 6.42321\times 10^{-5}\ \frac{\text{m}}{\text{s}}$$

(d) The Reynolds number can be computed as

$$\text{Re} = \frac{\rho d v}{\mu} = \frac{1000(10^{-5})6.42321\times 10^{-5}}{0.0014} = 0.0004588$$

This is far below 1, so the flow is very laminar.

(e) Before implementing Euler's method, the parameters can be substituted into the differential equation to give

$$\frac{dv}{dt} = 9.81\left(1 - \frac{1000}{2650}\right) - \frac{18(0.0014)}{2650(0.00001)^2} v = 6.108113 - 95{,}094v$$

The first two steps for Euler's method are

$$v(3.8147\times 10^{-6}) = 0 + (6.108113 - 95{,}094(0)) \times 3.8147\times 10^{-6} = 2.33006\times 10^{-5}$$

$$v(7.6294\times 10^{-6}) = 2.33006\times 10^{-5} + (6.108113 - 95{,}094(2.33006\times 10^{-5})) \times 3.8147\times 10^{-6} = 3.81488\times 10^{-5}$$

The remaining steps can be computed in a similar fashion as tabulated and plotted below:

t	v	dv/dt	t	v	dv/dt
0	0	6.108113	2.29×10^{-5}	5.99E-05	0.409017
3.81×10^{-6}	2.33E-05	3.892358	2.67×10^{-5}	6.15E-05	0.260643
7.63×10^{-6}	3.81E-05	2.480381	3.05×10^{-5}	6.25E-05	0.166093
1.14×10^{-5}	4.76E-05	1.580608	3.43×10^{-5}	6.31E-05	0.105842
1.53×10^{-5}	5.36E-05	1.007233	3.81×10^{-5}	6.35E-05	0.067447
1.91×10^{-5}	5.75E-05	0.641853			

1.23 (a) A force balance on a sphere can be written as:

$$m\frac{dv}{dt} = mg - \rho Vg - \frac{1}{2}\rho v|v| AC_d$$

(b) Dividing by mass gives

$$\frac{dv}{dt} = g - \frac{-\rho Vg}{m} - \frac{\rho AC_d}{2m} v|v|$$

The mass of the sphere is $\rho_s V$ where V = volume (m^3). The area and volume of a sphere are $\pi d^2/4$ and $\pi d^3/6$, respectively. Substituting these relationships gives

$$\frac{dv}{dt} = g\left(1 - \frac{\rho}{\rho_s}\right) - \frac{3\rho C_d}{4\rho_s d} v|v|$$

(c) At steady state, for a sphere falling downward

$$0 = g\left(1 - \frac{\rho}{\rho_s}\right) - \frac{3\rho C_d}{4\rho_s d} v^2$$

which can be solved for

$$v = \sqrt{\frac{4g\rho_s d}{3\rho C_d}\left(1 - \frac{\rho}{\rho_s}\right)}$$

Substituting the parameters gives

$$v = \sqrt{\frac{4(9.81)2700(0.01)}{3(1000)0.47}\left(1 - \frac{1000}{2700}\right)} = 0.68783\frac{\text{m}}{\text{s}}$$

(d) Before implementing Euler's method, the parameters can be substituted into the differential equation to give

$$\frac{dv}{dt} = 9.81\left(1 - \frac{1000}{2700}\right) - \frac{3(1000)0.47}{4(2700)(0.01)} v^2 = 6.176667 - 13.055556v^2$$

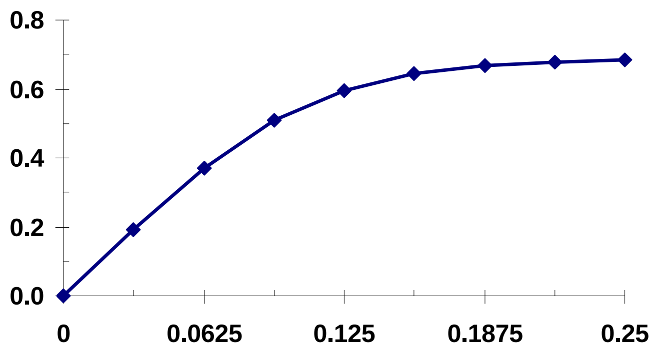
The first two steps for Euler's method are

$$v(0.03125) = 0 + (6.176667 - 13.055556(0)^2)0.03125 = 0.193021$$

$$v(0.0625) = 0.193021 + (6.176667 - 13.055556(0.193021)^2)0.03125 = 0.370841$$

The remaining steps can be computed in a similar fashion as tabulated and plotted below:

t	*v*	*dv/dt*	*t*	*v*	*dv/dt*
0	0.000000	6.176667	0.15625	0.643887	0.763953
0.03125	0.193021	5.690255	0.1875	0.667761	0.355136
0.0625	0.370841	4.381224	0.21875	0.678859	0.160023
0.09375	0.507755	2.810753	0.25	0.683860	0.071055
0.125	0.595591	1.545494			



1.24 Substituting the parameters into the differential equation gives

$$\frac{dy}{dx} = \frac{10000}{24(2\times10^{11})0.000325}\left(4x^3 - 12(4)x^2 + 12(4)^2 x\right)$$
$$= 2.5641\times10^{-5}\left(x^3 - 12x^2 + 48x\right)$$

The first step of Euler's method is

$$\frac{dy}{dx} = 2.5641\times10^{-5}\left((0)^3 - 12(0)^2 + 48(0)\right) = 0$$
$$y(0.125) = 0 + 0(0.125) = 0$$

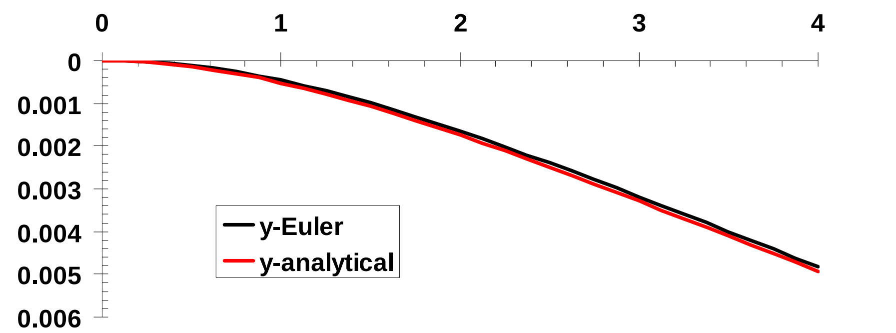
The second step is

$$\frac{dy}{dx} = 2.5641\times10^{-5}\left((0.125)^3 - 12(0.125)^2 + 48(0.125)\right) = 0.000149$$
$$y(0.25) = 0 + 0.000149(0.125) = 1.86361\times10^{-5}$$

The remainder of the calculations along with the analytical solution are summarized in the following table and plot. Note that the results of the numerical and analytical solutions are close.

x	*y*-Euler	*dy/dx*	*y*-analytical	*x*	*y*-Euler	*dy/dx*	*y*-analytical
0	0	0	0	2.125	0.001832	0.001472	0.001925
0.125	0	0.000149	9.42E-06	2.25	0.002016	0.001504	0.002111
0.25	1.86E-05	0.000289	3.69E-05	2.375	0.002204	0.001531	0.002301
0.375	5.47E-05	0.00042	8.13E-05	2.5	0.002395	0.001554	0.002494

0.5	0.000107	0.000542	0.000141	2.625	0.00259	0.001574	0.00269
0.625	0.000175	0.000655	0.000216	2.75	0.002787	0.001591	0.002887
0.75	0.000257	0.000761	0.000305	2.875	0.002985	0.001605	0.003087
0.875	0.000352	0.000859	0.000406	3	0.003186	0.001615	0.003288
1	0.000459	0.000949	0.000519	3.125	0.003388	0.001624	0.003491
1.125	0.000578	0.001032	0.000643	3.25	0.003591	0.00163	0.003694
1.25	0.000707	0.001108	0.000777	3.375	0.003795	0.001635	0.003898
1.375	0.000845	0.001177	0.00092	3.5	0.003999	0.001638	0.004103
1.5	0.000992	0.00124	0.001071	3.625	0.004204	0.00164	0.004308
1.625	0.001147	0.001298	0.00123	3.75	0.004409	0.001641	0.004513
1.75	0.00131	0.001349	0.001395	3.875	0.004614	0.001641	0.004718
1.875	0.001478	0.001395	0.001567	4	0.004819	0.001641	0.004923
2	0.001653	0.001436	0.001744				



1.25 [Note that students can easily get the underlying equations for this problem off the web]. The volume of a sphere can be calculated as

$$V_s = \frac{4}{3}\pi r^3$$

The portion of the sphere above water (the "cap") can be computed as

$$V_a = \frac{\pi h^2}{3}(3r - h)$$

Therefore, the volume below water is

$$V_s = \frac{4}{3}\pi r^3 - \frac{\pi h^2}{3}(3r - h)$$

Thus, the steady-state force balance can be written as

$$\rho_s g \frac{4}{3}\pi r^3 - \rho_f g \left[\frac{4}{3}\pi r^3 - \frac{\pi h^2}{3}(3r - h)\right] = 0$$

Cancelling common terms gives

$$\rho_s \frac{4}{3} r^3 - \rho_f \left[\frac{4}{3} r^3 - \frac{h^2}{3}(3r - h)\right] = 0$$

Collecting terms yields

$$\frac{\rho_f}{3}h^3 - r\rho_f h^2 - \left(\rho_s - \rho_f\right)\frac{4}{3}r^3 = 0$$

1.26 [Note that students can easily get the underlying equations for this problem off the web]. The total volume of a right circular cone can be calculated as

$$V_t = \frac{1}{3}\pi r_2^2 H$$

The volume of the frustum below the earth's surface can be computed as

$$V_b = \frac{\pi\left(H - h_1\right)}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

Archimedes' principle says that, at steady state, the downward force of the whole cone must be balanced by the upward buoyancy force of the below ground frustum,

$$\frac{1}{3}\pi r_2^2 Hg\rho_g = \frac{\pi\left(H - h_1\right)}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)g\rho_b \qquad (1)$$

Before proceeding we have too many unknowns: r_1 and h_1. So before solving, we must eliminate r_1 by recognizing that using similar triangles ($r_1/h_1 = r_2/H$)

$$r_1 = \frac{r_2}{H}h_1$$

which can be substituted into Eq. (1) (and cancelling the g's)

$$\frac{1}{3}\pi r_2^2 H\rho_g = \frac{\pi\left(H - h_1\right)}{3}\left(\left(\frac{r_2}{H}h_1\right)^2 + r_2^2 + \frac{r_2^2}{H}h_1\right)\rho_b$$

Therefore, the equation now has only 1 unknown: h_1, and the steady-state force balance can be written as

$$\rho_s g\frac{4}{3}\pi r^3 - \rho_f g\left[\frac{4}{3}\pi r^3 - \frac{\pi h^2}{3}\left(3r - h\right)\right] = 0$$

Cancelling common terms gives

$$\rho_s\frac{4}{3}r^3 - \rho_f\left[\frac{4}{3}r^3 - \frac{h^2}{3}\left(3r - h\right)\right] = 0$$

and collecting terms yields

$$\frac{\rho_f}{3}h^3 - r\rho_f h^2 - \left(\rho_s - \rho_f\right)\frac{4}{3}r^3 = 0$$