Stewart - Calculus ET 8e Chapter 11 Form A

1. Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

   $$\left\{-\frac{1}{2}, \frac{16}{3}, -\frac{81}{4}, \frac{256}{5}, -\frac{625}{6}, \ldots\right\}$$

2. Find the partial sum S_7 of the series $\sum_{n=1}^{\infty} \frac{6}{10+8^n}$. Give your answer to five decimal places.

3. How many terms of the series $\sum_{n=2}^{\infty} \frac{12}{6n(\ln n)^2}$ would you need to add to find its sum to within 0.02?

4. Test the series for convergence or divergence.

   $$\sum_{k=5}^{\infty} \frac{5}{k(\ln k)^7}$$

5. Use the sum of the first 10 terms to approximate the sum of the series. Estimate the error.

   $$\sum_{n=1}^{\infty} \frac{1}{1+4^n}$$

6. Test the series for convergence or divergence.

   $$\sum_{n=2}^{\infty} (-1)^n \frac{n}{5\ln n}$$

7. Test the series for convergence or divergence.

   $$\sum_{n=1}^{\infty} (-4)^n \frac{\ln n}{\sqrt{n}}$$

8. Use the binomial series to expand the function as a power series. Find the radius of convergence.

   $$\frac{1}{(4+x)^5}$$

9. Find the sum of the series.

$$\sum_{n=0}^{\infty} \frac{2^n}{3^n n!}$$

10. Use the Alternating Series Estimation Theorem or Taylor's Inequality to estimate the range of values of *x* for which the given approximation is accurate to within the stated error.

$$\cos x \approx 1 - \frac{x^2}{2} + \frac{x^4}{24} \quad |error| < 0.08$$

Write *a* such that $-a < x < a$.

11. Write the first five terms of the sequence $\{a_n\}$ whose n^{th} term is given.

$$a_n = \frac{n+7}{6n-1}$$

12. Find an expression for the n^{th} term of the sequence. (Assume that the pattern continues.)

$$\left\{\frac{2}{25}, \frac{4}{36}, \frac{6}{49}, \frac{8}{64}, \frac{10}{81}, \ldots\right\}$$

13. Determine whether the given series converges or diverges. If it converges, find its sum.

$$\sum_{n=0}^{\infty} \frac{9^n + 8^n}{12^n}$$

14. Determine whether the given series is convergent or divergent.

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

15. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{2^n}$$

16. Test the series for convergence or divergence.

$$\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^5+8}}$$

17. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n+4}$$

18. Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{(n!)^4}{(7n)!}$$

19. Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{9^n}{n!\,n}$$

20. Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \left(\frac{\ln\left(n^6\right)}{n}\right)^n$$

Answer Key

1. $a_n = \dfrac{(-1)^n n^4}{n+1}$
2. 0.42758
3. $m > e^{100}$
4. convergent
5. 0.27940, error < 0.0000007
6. divergent
7. divergent
8. $|x| < 4$
9. $e^{2/3}$
10. $-1.965 < x < 1.965$
11. $\dfrac{8}{5}, \dfrac{9}{11}, \dfrac{10}{17}, \dfrac{11}{23}, \dfrac{12}{29}$
12. $a_n = \dfrac{2n}{(n+4)^2}$
13. 7
14. Convergent
15. Converges
16. Convergent
17. Converges
18. converges
19. convergent
20. convergent

Stewart - Calculus ET 8e Chapter 11 Form B

1. Determine whether the sequence converges or diverges. If it converges, find the limit.

   $a_n = 2e^{4n/(n+2)}$

2. Find the exact value of the limit of the sequence defined by $a_1 = \sqrt{4}$, $a_{n+1} = \sqrt{4+a_n}$.

3. The terms of a series are defined recursively by the equations $a_1 = 6$, $a_{n+1} = \dfrac{7n+1}{6n+3}a_n$.

   Determine whether $\sum a_n$ converges or diverges.

4. Express the number $0.\overline{81}$ as a ratio of integers.

5. Use the Integral Test to determine whether the series is convergent or divergent.

   $$\sum_{n=1}^{\infty} \frac{1}{8n+2}$$

6. How many terms of the series $\sum_{m=2}^{\infty} \dfrac{12}{6m(\ln m)^2}$ would you need to add to find its sum to within 0.02?

7. Test the series for convergence or divergence.

   $$\sum_{k=5}^{\infty} \frac{5}{k(\ln k)^7}$$

8. Determine whether the sequence convergent or divergent.

   $$\sum_{n=1}^{\infty} \frac{1}{n^2-6n+10}$$

9. Test the series for convergence or divergence.

   $$\sum_{k=1}^{\infty} \frac{(-6)^{k+1}}{7^{2k}}$$

Stewart - Calculus ET 8e Chapter 11 Form B

10. Test the series for convergence or divergence.

$$\sum_{m=1}^{\infty} \frac{4^m m^3}{m!}$$

11. Find a power series representation for the function and determine the radius of convergence.

$$f(x) = \arctan\left(\frac{x}{3}\right)$$

12. Find the Maclaurin series for $f(x)$ using the definition of a Maclaurin serires.

$$f(x) = (3+x)^{-3}$$

13. Use the binomial series to expand the function as a power series. Find the radius of convergence.

$$\sqrt[4]{1+x^6}$$

14. Use the Alternating Series Estimation Theorem or Taylor's Inequality to estimate the range of values of *x* for which the given approximation is accurate to within the stated error.

$$\cos x \approx 1 - \frac{x^2}{2} + \frac{x^4}{24} \quad |error| < 0.08$$

Write *a* such that $-a < x < a$.

15. Use the sum of the first 9 terms to approximate the sum of the following series.

$$\sum_{n=1}^{\infty} \frac{6}{n^7 + n^2}$$

Write your answer to six decimal places.

16. Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n\sqrt{n+7}}$$

17. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} (-1)^n n \sin\left(\frac{\pi}{9n}\right)$$

18. Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n\sqrt{n+6}}$$

19. Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{(n!)^4}{(7n)!}$$

20. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[6]{n}}$$

<u>Stewart - Calculus ET 8e Chapter 11 Form B</u>

Answer Key

1. $2e^{4}$
2. $\dfrac{1+\sqrt{17}}{2}$
3. diverges
4. $\dfrac{9}{11}$
5. divergent
6. $m > e^{100}$
7. convergent
8. converges
9. convergent
10. convergent
11. $\displaystyle\sum_{n=0}^{\infty}(-1)^{n}\frac{\left(\frac{x}{3}\right)^{2n+1}}{2n+1}$; $R = 3$
12. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^{n}(n+1)(n+2)\left(\frac{x}{3}\right)^{n}}{54}$
13. $|x| < 1$
14. $-1.965 < x < 1.965$
15. 3.048662
16. Convergent
17. Diverges
18. Convergent
19. converges
20. conditionally convergent

Stewart – Calculus ET 8e Chapter 11 Form C

Select the correct answer for each question.

____ 1. Determine whether the sequence defined by $a_n = \dfrac{n^2 - 5}{6n^2 + 1}$ converges or diverges. If it converges, find its limit.

a. $\dfrac{1}{6}$

b. –5

c. $-\dfrac{5}{6}$

d. Diverges

____ 2. Determine whether the sequence defined by $a_n = \dfrac{5^n}{8^n + 1}$ converges or diverges. If it converges, find its limit.

a. 1

b. $\dfrac{5}{8}$

c. 0

d. Diverges

____ 3. Find the value of the limit for the sequence given.

$$\left\{ \frac{1 \cdot 9 \cdot 17 \cdots (7n+1)}{(7n)^2} \right\}$$

a. 0
b. -1
c. π
d. 3
e. 1

____ 4. If \$600 is invested at 4% interest, compounded annually, then after *n* years the investment is worth $a_n = 600(1.04)^n$ dollars. Find the size of investment after 7 years.

a. \$430.21
b. \$1,860.81
c. \$1,230.81
d. \$789.56
e. \$1,321.06

____ 5. Determine whether the geometric series converges or diverges. If it converges, find its sum.

$$\sum_{n=0}^{\infty} 5^n 6^{-n+1}$$

a. 30
b. 36
c. 5
d. Diverges

____ 6. A sequenceis $\{a_n\}$ defined recursively by the equation $a_n = 0.5(a_{n-1} + a_{n-2})$ for $n \geq 3$ where $a_1 = 14, a_2 = 14$.

Use your calculator to guess the limit of the sequence.

a. 6
b. 14
c. 26
d. 17
e. 15

Stewart – Calculus ET 8e Chapter 11 Form C

____ 7. Determine whether the geometric series converges or diverges. If it converges, find its sum.

$-\frac{1}{5}+\frac{1}{25}-\frac{1}{125}+\frac{1}{625}-\cdots$

a. $\frac{1}{4}$

b. $-\frac{1}{5}$

c. Diverges

d. $-\frac{1}{6}$

____ 8. Find all positive values of *u* for which the series $\sum_{n=1}^{\infty} 6u^{\ln 7n}$ converges.

a. $u > 7$

b. $6 < u < \frac{7}{e}$

c. $0 < u < \frac{1}{e}$

d. $u < 6$

e. $u > \ln 7$

____ 9. Determine which one of the *p*-series below is divergent.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{0.3}}$

b. $\displaystyle\sum_{n=1}^{\infty} n^{-4\pi}$

c. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3e}}$

d. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{4}}$

____ 10. Find an approximation of the sum of the series accurate to two decimal places.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$$

a. –1.06
b. –0.84
c. –0.90
d. –0.98

____ 11. Approximate the sum to the indicated accuracy.

$$\sum_{n=1}^{\infty} \frac{4(-1)^{n-1}}{n^7}$$ (five decimal places)

a. 6.97036
b. 4.97036
c. 7.97036
d. 3.97036
e. 5.97036

____ 12. Find the radius of convergence and the interval of convergence of the power series.

$$\sum_{n=0}^{\infty} \frac{(7x)^n}{n!}$$

a. $R = 7,\ I = (-7,\ 7)$
b. $R = 0,\ I = \{0\}$
c. $R = 7,\ I = [-7,\ 7]$
d. $R = \infty,\ I = (-\infty,\ \infty)$

____ 13. Find the radius of convergence and the interval of convergence of the power series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-8)^n}{\sqrt{n}}$$

a. $R = 1,\ I = [7,\ 9)$
b. $R = 1,\ I = (7,\ 9]$
c. $R = 8,\ I = [-8,\ 8)$
d. $R = 8,\ I = (-8,\ 8)$

____ 14. Find the interval of convergence of the series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n+3}$$

a. $[-1, 1]$
b. $(-1, 1)$
c. $(-1, 1]$
d. diverges everywhere
e. $[-1, 1)$

____ 15. Find a power series representation for

$f(t) = \ln(14 - t)$

a. $\ln 14 - \sum_{n=1}^{\infty} \frac{t^n}{14^n}$

b. $\ln 14 - \sum_{n=1}^{\infty} \frac{t^n}{n14^n}$

c. $\sum_{n=0}^{\infty} \frac{t^n}{n14^n}$

d. $\sum_{n=1}^{\infty} \frac{14t^n}{n^n}$

e. $\ln 14 + \sum_{n=1}^{\infty} \frac{t^{2n}}{14^n}$

____ 16. Use the power series for $f(x) = \sqrt[3]{5+x}$ to estimate $\sqrt[3]{5.07}$ correct to four decimal places.

a. 1.7179
b. 1.7189
c. 1.7195
d. 1.7156
e. 1.7200

____ 17. Use series to approximate the definite integral to within the indicated accuracy.

$\int_0^{0.5} x^2 e^{-x^2}\,dx \quad |error| < 0.001$

a. 0.0354
b. 0.0125
c. 0.0625
d. 0.1447
e. 0.2774

____ 18. Use series to evaluate the limit correct to three decimal places.

$$\lim_{x \to 0} \frac{7x - \tan^{-1} 7x}{x^3}$$

Select the correct answer.

a. 118.933
b. 114.133
c. 34.3233
d. 114.333
e. 115.933

____ 19. For which positive integers k is the series $\sum_{n=1}^{\infty} \frac{(n!)^5}{(kn)!}$ convergent?

a. $k \geq 5$
b. $k \leq 0$
c. $k \geq 0$
d. $k \geq 1$
e. $k \leq -5$

____ 20. Which of the given series are absolutely convergent?

a. $\sum_{n=1}^{\infty} \frac{\sin 2n}{n}$

b. $\sum_{n=1}^{\infty} \frac{\cos \frac{\pi n}{7}}{n\sqrt{n}}$

Stewart – Calculus ET 8e Chapter 11 Form C

Answer Key

1. A
2. C
3. A
4. D
5. B
6. B
7. D
8. C
9. A
10. C
11. D
12. D
13. B
14. C
15. B
16. A
17. A
18. D
19. A
20. B

Stewart - Calculus ET 8e Chapter 11 Form D

Select the correct answer for each question.

____ 1. Find the value of the limit for the sequence given.

$$\left\{ \frac{1 \cdot 9 \cdot 17 \cdots (7n+1)}{(7n)^2} \right\}$$

a. 0
b. -1
c. π
d. 3
e. 1

____ 2. Determine whether the sequence defined by $a_n = 5 + 8(-1)^n$ converges or diverges. If it converges, find its limit.

a. 13
b. 5
c. Diverges
d. –3

____ 3. Determine whether the geometric series converges or diverges. If it converges, find its sum.

$$\sum_{n=0}^{\infty} 3^n 4^{-n+1}$$

a. 12
b. Diverges
c. 3
d. 16

____ 4. Determine whether the geometric series converges or diverges. If it converges, find its sum.

$$\sum_{n=0}^{\infty} 5^n 6^{-n+1}$$

a. 30
b. 36
c. 5
d. Diverges

____ 5. A rubber ball is dropped from a height of 8 m onto a flat surface. Each time the ball hits the surface, it rebounds to 50% of its previous height. Find the total distance the ball travels.

a. 16
b. 24
c. 8
d. 32

____ 6. A sequenceis $\{a_n\}$ defined recursively by the equation $a_n = 0.5(a_{n-1} + a_{n-2})$ for $n \geq 3$ where $a_1 = 14, a_2 = 14.$

Use your calculator to guess the limit of the sequence.

a. 6
b. 14
c. 26
d. 17
e. 15

____ 7. Find the sum of the series.

$$\frac{2}{1 \cdot 3} - \frac{2^2}{2 \cdot 3^2} + \frac{2^3}{3 \cdot 3^3} - \frac{2^4}{4 \cdot 3^4} + \ldots$$

a. $\ln\left(\frac{4}{3}\right)$

b. $\frac{5e}{3}$

c. $\ln\left(\frac{5}{3}\right)$

d. $\ln\left(\frac{1}{3}\right)$

e. $e^{5/3}$

____ 8. Find all positive values of *u* for which the series $\sum_{n=1}^{\infty} 6u^{\ln 7n}$ converges.

a. $u > 7$

b. $6 < u < \frac{7}{e}$

c. $0 < u < \frac{1}{e}$

d. $u < 6$

e. $u > \ln 7$

____ 9. Find all values of *p* for which the series $\sum_{n=1}^{\infty} \frac{\ln(n^9)}{n^p}$ converges.

a. $p < 9$
b. $p < 1$
c. $p > 9$
d. $p > 1$

____ 10. Determine whether the sequence convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{3}{n^2+3}$$

a. converges
b. diverges

____ 11. Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{(-6)^{n+1}}{4^{8n}}$$

a. The series is convergent.
b. The series is divergent.

____ 12. Determine which series is convergent.

a. $-\frac{2}{7}+\frac{3}{8}-\frac{4}{9}+\frac{5}{10}-\frac{6}{11}-\ldots$

b. $\frac{4}{3}-\frac{4}{4}+\frac{4}{5}-\frac{4}{6}+\frac{4}{7}-\ldots$

____ 13. Find the values of p for which the series is convergent.

$$\sum_{n=2}^{\infty}\frac{(-1)^n}{\left(\ln\left(n^6\right)\right)^p}$$

a. $p>1$
b. $p>0$
c. $p<0$
d. $p<1$

____ 14. Find the radius of convergence and the interval of convergence of the power series.

$$\sum_{n=0}^{\infty}\frac{(7x)^n}{n!}$$

a. $R=7,\ I=(-7,\ 7)$
b. $R=0,\ I=\{0\}$
c. $R=7,\ I=[-7,\ 7]$
d. $R=\infty,\ I=(-\infty,\ \infty)$

____ 15. Suppose that the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$ is 9. What is the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^{2n}$.

a. 252
b. 3
c. 1
d. 256
e. 16

____ 16. Find the radius of convergence and the interval of convergence of the power series.

$$\sum_{n=0}^{\infty}\left(\frac{nx}{6}\right)^n$$

a. $R = 0,\ I = \{0\}$
b. $R = \infty,\ I = (-\infty, \infty)$
c. $R = 6,\ I = [-6, 6]$
d. $R = 6,\ I = (-6, 6)$

____ 17. Find the Maclaurin series for $f(x)$ using the definition of the Maclaurin series.

$f(x) = x\cos(4x)$

a. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n 4^n x^{2n+1}}{(2n)!}$

b. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n 4^{2n} x^{2n+1}}{n!}$

c. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n 4^{2n} x^{2n}}{(2n)!}$

d. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n 4^{2n} x^{2n+1}}{(2n)!}$

e. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^{n+1} 4^{2n} x^{2n+1}}{(2n)!}$

____ 18. Use multiplication or division of power series to find the first three nonzero terms in the Maclaurin series for the function.

$f(x) = 5e^{-x^2}\cos 4x$

a. $5\left(1 - 17x^2 + \dfrac{115}{6}x^4\right)$

b. $5\left(1 - 9x^2 + \dfrac{115}{6}x^4\right)$

c. $5\left(1 - 9x + \dfrac{115}{6}x^4\right)$

d. $5\left(1 - 9x^2 + \dfrac{97}{6}x^4\right)$

e. $5\left(1 - 17x^2 + \dfrac{67}{6}x^4\right)$

____ 19. Given the series $\sum_{m=1}^{\infty} \dfrac{3m}{4^m(3m+5)}$ estimate the error in using the partial sum s_8 by comparison with the series $\sum_{m=9}^{\infty} \dfrac{1}{4^m}$.

a. $R_8 \le 2.6130051$
b. $R_8 \ge 0.0000052$
c. $R_8 \le 0.0000051$
d. $R_8 \ge 0.0000051$
e. $R_8 \le 0.000005$

____ 20. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^n \arctan n}{n^4}$$

a. conditionally convergent
b. absolutely convergent
c. divergent

Stewart - Calculus ET 8e Chapter 11 Form D

Answer Key

1. A
2. C
3. D
4. B
5. B
6. B
7. C
8. C
9. D
10. A
11. A
12. B
13. B
14. D
15. B
16. A
17. D
18. B
19. C
20. B

1. Determine whether the sequence defined by $a_n = \dfrac{n^2 - 5}{6n^2 + 1}$ converges or diverges. If it converges, find its limit.

____ 2. Determine whether the sequence defined by $a_n = 5 + 8(-1)^n$ converges or diverges. If it converges, find its limit. Select the correct answer.

   a. 13
   b. 5
   c. Diverges
   d. –3

3. Determine whether the sequence defined by $a_n = \dfrac{\sin 2n}{9n}$ converges or diverges. If it converges, find its limit.

4. Determine whether the series is convergent or divergent by expressing S_n as a telescoping sum. If it is convergent, find its sum.

$$\sum_{n=2}^{\infty} \frac{5}{n\left(n^2 - 1\right)}.$$

5. Determine whether the geometric series converges or diverges. If it converges, find its sum.

$$\sum_{n=0}^{\infty} 5^n 6^{-n+1}$$

____ 6. A sequenceis $\left\{a_n\right\}$ defined recursively by the equation $a_n = 0.5\left(a_{n-1} + a_{n-2}\right)$ for $n \geq 3$ where $a_1 = 14, a_2 = 14.$

   Use your calculator to guess the limit of the sequence. Select the correct answer.

   a. 6
   b. 14
   c. 26
   d. 17
   e. 15

7. Determine which one of the *p*-series below is convergent.

8. Determine which one of the *p*-series below is divergent.

9. Let $a_k = f(k)$, where *f* is a continuous, positive, and decreasing function on $[n, \infty)$, and suppose that $\sum_{k=1}^{\infty} a_k$ is convergent. Defining $R_n = S - S_n$, where $S = \sum_{n=1}^{\infty} a_n$ and $S_n = \sum_{k=1}^{n} a_k$, we have that $\int_{n+1}^{\infty} f(x)dx \le R_n \le \int_{n}^{\infty} f(x)dx$. Find the maximum error if the sum of the series $\sum_{n=1}^{\infty} \frac{3}{n^2}$ is approximated by S_{40}.

____ 10. Test the series for convergence or divergence. Select the correct answer.

$$\sum_{m=1}^{\infty} \frac{(-6)^{m+1}}{4^{8m}}$$

a. The series is convergent.
b. The series is divergent.

11. Approximate the sum to the indicated accuracy.

$$\sum_{n=1}^{\infty} \frac{4(-1)^{n-1}}{n^7}$$ (five decimal places)

12. Find the radius of convergence and the interval of convergence of the power series.

$$\sum_{n=0}^{\infty} \frac{(7x)^n}{n!}$$

13. Find the radius of convergence and the interval of convergence of the power series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-8)^n}{\sqrt{n}}$$

Stewart - Calculus ET 8e Chapter 11 Form E

____ 14. Find the radius of convergence and the interval of convergence of the power series. Select the correct answer.

$$\sum_{n=2}^{\infty} \frac{x^n}{n(\ln n)^8}$$

a. $R = 0,\ I = \{0\}$
b. $R = 1,\ I = [-1,\ 1]$
c. $R = 1,\ I = (-1,\ 1)$
d. $R = \infty,\ I = (-\infty,\ \infty)$

15. Use the power series for $f(x) = \sqrt[3]{5+x}$ to estimate $\sqrt[3]{5.07}$ correct to four decimal places.

16. Use series to evaluate the limit correct to three decimal places.

$$\lim_{x \to 0} \frac{7x - \tan^{-1} 7x}{x^3}$$

Select the correct answer.

17. Use the binomial series to expand the function as a power series. Find the radius of convergence.

$$\frac{x}{\sqrt{16+x^2}}$$

18. Given the series $\sum_{m=1}^{\infty} \frac{3m}{4^m(3m+5)}$ estimate the error in using the partial sum s_8 by comparison with the series $\sum_{m=9}^{\infty} \frac{1}{4^m}$.

____ 19. Determine whether the series is absolutely convergent, conditionally convergent, or divergent. Select the correct answer.

$$\sum_{n=1}^{\infty} \left(\frac{4n^2+3}{3n^2+4} \right)^n$$

a. conditionally convergent
b. absolutely convergent
c. divergent

20. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^n \arctan n}{n^4}$$

Stewart - Calculus ET 8e Chapter 11 Form E

Answer Key

1. $\frac{1}{6}$
2. C
3. 0
4. $\frac{5}{4}$
5. 36
6. B
7. $\sum_{n=1}^{\infty} \frac{1}{n^6}$
8. $\sum_{n=1}^{\infty} \frac{1}{n^{03}}$
9. 0.075
10. A
11. 3.97036
12. $R = \infty$, $I = (-\infty, \infty)$
13. $R = 1$, $I = (7, 9]$
14. B
15. 1.7179
16. 114.333
17. $|x| < 4$
18. $R_8 \leq 0.0000051$
19. C
20. absolutely convergent

Stewart - Calculus ET 8e Chapter 11 Form F

____ 1. Determine whether the sequence defined by $a_n = \dfrac{n^2 - 5}{6n^2 + 1}$ converges or diverges. If it converges, find its limit. Select the correct answer.

a. $\dfrac{1}{6}$

b. –5

c. $-\dfrac{5}{6}$

d. Diverges

2. Determine whether the sequence converges or diverges. If it converges, find the limit.

$a_n = e^{n/(n+6)}$

____ 3. Find the value of the limit for the sequence given. Select the correct answer.

$$\left\{ \frac{1 \cdot 9 \cdot 17 \cdots (7n+1)}{(7n)^2} \right\}$$

a. 0
b. -1
c. π
d. 3
e. 1

4. If $600 is invested at 4% interest, compounded annually, then after *n* years the investment is worth $a_n = 600(1.04)^n$ dollars. Find the size of investment after 7 years.

5. A sequenceis $\{a_n\}$ defined recursively by the equation $a_n = 0.5(a_{n-1} + a_{n-2})$ for $n \geq 3$ where $a_1 = 14, a_2 = 14$.

Use your calculator to guess the limit of the sequence.

6. Determine whether the geometric series converges or diverges. If it converges, find its sum.
$-\dfrac{1}{5} + \dfrac{1}{25} - \dfrac{1}{125} + \dfrac{1}{625} - \cdots$

____ 7. When money is spent on goods and services, those that receive the money also spend some of it. The people receiving some of the twice-spent money will spend some of that, and so on. Economists call this chain reaction the multiplier effect. In a hypothetical isolated community, the local government begins the process by spending *D* dollars. Suppose that each recipient of spent money spends $100c\%$ and saves $100s\%$ of the money that he or she receives. The values *c* and *s* are called the marginal propensity to consume and the marginal propensity to save and, of course, $c+s=1$.

The number $k = 1/s$ is called the multiplier. What is the multiplier if the marginal propensity to consume is 90%?

Select the correct answer.

a. 4
b. 3
c. 6
d. 7
e. 10

8. Find the sum of the series.

$$\frac{2}{1\cdot 3}-\frac{2^2}{2\cdot 3^2}+\frac{2^3}{3\cdot 3^3}-\frac{2^4}{4\cdot 3^4}+\ldots$$

9. Find an approximation of the sum of the series accurate to two decimal places.

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}$$

____ 10. Determine which series is convergent. Select the correct answer.

a. $-\frac{2}{7}+\frac{3}{8}-\frac{4}{9}+\frac{5}{10}-\frac{6}{11}-\ldots$

b. $\frac{4}{3}-\frac{4}{4}+\frac{4}{5}-\frac{4}{6}+\frac{4}{7}-\ldots$

11. Approximate the sum to the indicated accuracy.

$$\sum_{n=1}^{\infty}\frac{4(-1)^{n-1}}{n^7}$$ (five decimal places)

12. Find the radius of convergence and the interval of convergence of the power series.

$$\sum_{n=2}^{\infty} \frac{x^n}{n(\ln n)^8}$$

____ 13. Find the radius of convergence and the interval of convergence of the power series. \
Select the correct answer.

$$\sum_{n=1}^{\infty} \frac{3 \cdot 6 \cdot 9 \cdot \cdots \cdot 3n}{4 \cdot 7 \cdot 10 \cdot \cdots \cdot (3n+1)} x^{2n+1}$$

a. $R = \infty, I = (-\infty, \infty)$
b. $R = 1, I = (-1, 1)$
c. $R = 0, I = \{0\}$
d. $R = 1, I = [-1, 1]$

14. Suppose that the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$ is 9. What is the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^{2n}$.

15. Find the radius of convergence and the interval of convergence of the power series.

$$\sum_{n=0}^{\infty} \frac{x^n}{n+2}$$

____ 16. Find the radius of convergence of the series. Select the correct answer.

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x+10)^n}{n 6^n}$$

a. $(-8, 6]$
b. $(2, 14]$
c. $(-14, -2)$
d. $[-16, -4)$
e. $[-1, 1]$

17. Use series to evaluate the limit correct to three decimal places.

$$\lim_{x \to 0} \frac{7x - \tan^{-1} 7x}{x^3}$$

Select the correct answer.

18. Given the series $\sum_{m=1}^{\infty} \frac{3m}{4^m(3m+5)}$ estimate the error in using the partial sum s_8 by comparison with the series $\sum_{m=9}^{\infty} \frac{1}{4^m}$.

____ 19. For which positive integers k is the series $\sum_{n=1}^{\infty} \frac{(n!)^5}{(kn)!}$ convergent? Select the correct answer.

a. $k \ge 5$
b. $k \le 0$
c. $k \ge 0$
d. $k \ge 1$
e. $k \le -5$

____ 20. Which of the given series are absolutely convergent? Select the correct answer.

a. $\sum_{n=1}^{\infty} \frac{\sin 2n}{n}$

b. $\sum_{n=1}^{\infty} \frac{\cos \frac{\pi n}{7}}{n\sqrt{n}}$

Answer Key

1. A
2. e
3. A
4. $789.56
5. 14
6. $-\frac{1}{6}$
7. E
8. $\ln\left(\frac{5}{3}\right)$
9. –0.90
10. B
11. 3.97036
12. $R = 1,\ I = [-1,\ 1]$
13. B
14. 3
15. $R = 1,\ I = [-1,\ 1)$
16. D
17. 114.333
18. $R_8 \le 0.0000051$
19. A
20. B

Stewart - Calculus ET 8e Chapter 11 Form G

____ 1. Find the value of the limit of the sequence defined by

$a_1 = 1,\ a_{n+1} = 6 - \dfrac{1}{a_n}.$

Select the correct answer.

a. $\dfrac{6+\sqrt{5}}{2}$

b. $\dfrac{6-\sqrt{10}}{2}$

c. $\dfrac{6+\sqrt{10}}{2}$

d. $\dfrac{6-\sqrt{5}}{2}$

e. $6+\sqrt{10}$

2. Determine whether the sequence converges or diverges. If it converges, find the limit.

$a_n = e^{n/(n+6)}$

3. Determine whether the sequence defined by $a_n = 5 + 8(-1)^n$ converges or diverges. If it converges, find its limit.

____ 4. Determine whether the geometric series converges or diverges. If it converges, find its sum.

$\sum_{n=0}^{\infty} 3^n 4^{-n+1}$

Select the correct answer.

a. 12
b. Diverges
c. 3
d. 16

Stewart - Calculus ET 8e Chapter 11 Form G

5. Determine whether the given series converges or diverges. If it converges, find its sum.

$$\sum_{n=1}^{\infty}\left(1+\frac{5}{n}\right)^{n}$$

6. Determine whether the given series converges or diverges. If it converges, find its sum.

$$\sum_{n=0}^{\infty}\frac{9n^2+3}{2n^2+5}$$

7. Determine which one of the *p*-series below is convergent.

____ 8. How many terms of the series do we need to add in order to find the sum to the indicated accuracy? Select the correct answer.

$$\sum_{n=1}^{\infty} 2\frac{(-1)^{n+1}}{n^2} \quad \left(|\text{error}|\right) < 0.0798$$

a. $n = 6$
b. $n = 5$
c. $n = 12$
d. $n = 8$
e. $n = 13$

9. Find an approximation of the sum of the series accurate to two decimal places.

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}$$

10. Determine which series is convergent.

____ 11. Find the values of *p* for which the series is convergent. Select the correct answer.

$$\sum_{n=2}^{\infty}\frac{(-1)^n}{\left(\ln\left(n^6\right)\right)^p}$$

a. $p > 1$
b. $p > 0$
c. $p < 0$
d. $p < 1$

12. Find the radius of convergence and the interval of convergence of the power series.

$$\sum_{n=1}^{\infty} \frac{3 \cdot 6 \cdot 9 \cdot \cdots \cdot 3n}{4 \cdot 7 \cdot 10 \cdot \cdots \cdot (3n+1)} x^{2n+1}$$

13. Find the interval of convergence of the series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n+3}$$

14. Find the radius of convergence of the series.

$$\sum_{n=1}^{\infty} \frac{n^3 x^n}{2^n}$$

____ 15. Find the radius of convergence of the series. Select the correct answer.

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x+10)^n}{n 6^n}$$

a. $(-8, 6]$
b. $(2, 14]$
c. $(-14, -2)$
d. $[-16, -4)$
e. $[-1, 1]$

16. Find a power series representation for the function.

$$f(y) = \ln\left(\frac{11+y}{11-y}\right)$$

17. Find a power series representation for

$$f(t) = \ln(14 - t)$$

18. Find the Maclaurin series for $f(x)$ using the definition of the Maclaurin series.

$$f(x) = x\cos(4x)$$

19. For which positive integers k is the series $\sum_{n=1}^{\infty} \frac{(n!)^5}{(kn)!}$ convergent?

____ 20. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^n \arctan n}{n^4}$$

Select the correct answer.

a. conditionally convergent
b. absolutely convergent
c. divergent

Stewart - Calculus ET 8e Chapter 11 Form G

Answer Key

1. A
2. e
3. Diverges
4. D
5. Diverges
6. Diverges
7. $\sum_{n=1}^{\infty} \frac{1}{n^6}$
8. A
9. –0.90
10. $\frac{4}{3} - \frac{4}{4} + \frac{4}{5} - \frac{4}{6} + \frac{4}{7} - \ldots$
11. B
12. $R = 1,\ I = (-1,\ 1)$
13. $(-1, 1]$
14. $R = 2$
15. D
16. $\sum_{n=0}^{\infty} \frac{2y^{2n+1}}{11^{n+1}(2n+1)}$
17. $\ln 14 - \sum_{n=1}^{\infty} \frac{t^n}{n14^n}$
18. $\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n} x^{2n+1}}{(2n)!}$
19. $k \geq 5$
20. B

Stewart – Calculus ET 8e Chapter 11 Form H

____ 1. Find the value of the limit for the sequence given. Select the correct answer.

$$\left\{ \frac{1 \cdot 9 \cdot 17 \cdots (7n+1)}{(7n)^2} \right\}$$

a. 0
b. -1
c. π
d. 3
e. 1

2. If \$600 is invested at 4% interest, compounded annually, then after *n* years the investment is worth $a_n = 600(1.04)^n$ dollars. Find the size of investment after 7 years.

3. Determine whether the given series converges or diverges. If it converges, find its sum.

$$\sum_{n=1}^{\infty} \left(1 + \frac{5}{n}\right)^n$$

4. Determine whether the geometric series converges or diverges. If it converges, find its sum.

$$\sum_{n=0}^{\infty} 5^n 6^{-n+1}$$

____ 5. A rubber ball is dropped from a height of 8 m onto a flat surface. Each time the ball hits the surface, it rebounds to 50% of its previous height. Find the total distance the ball travels. Select the correct answer.

a. 16
b. 24
c. 8
d. 32

6. Determine whether the geometric series converges or diverges. If it converges, find its sum.

$$-\frac{1}{5} + \frac{1}{25} - \frac{1}{125} + \frac{1}{625} - \cdots$$

7. Determine whether the given series converges or diverges. If it converges, find its sum.

$$\sum_{n=0}^{\infty} \frac{9n^2+3}{2n^2+5}$$

____ 8. Find all positive values of *u* for which the series $\sum_{n=1}^{\infty} 6u^{\ln 7n}$ converges. Select the correct answer.

a. $u > 7$

b. $6 < u < \frac{7}{e}$

c. $0 < u < \frac{1}{e}$

d. $u < 6$

e. $u > \ln 7$

9. Find all values of *p* for which the series $\sum_{n=1}^{\infty} \frac{\ln\left(n^9\right)}{n^p}$ converges.

10. Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{(-6)^{n+1}}{4^{8n}}$$

11. Find an approximation of the sum of the series accurate to two decimal places.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$$

12. Approximate the sum to the indicated accuracy.

$\sum_{n=1}^{\infty} \frac{4(-1)^{n-1}}{n^7}$ (five decimal places)

____ 13. Find the radius of convergence and the interval of convergence of the power series.]
Select the correct answer.

$$\sum_{n=1}^{\infty} \frac{3 \cdot 6 \cdot 9 \cdot \cdots \cdot 3n}{4 \cdot 7 \cdot 10 \cdot \cdots \cdot (3n+1)} x^{2n+1}$$

a. $R = \infty,\ I = (-\infty, \infty)$
b. $R = 1,\ I = (-1, 1)$
c. $R = 0,\ I = \{0\}$
d. $R = 1,\ I = [-1, 1]$

14. Find the radius of convergence and the interval of convergence of the power series.

$$\sum_{n=0}^{\infty} \left(\frac{nx}{6}\right)^{n}$$

____ 15. Find the interval of convergence of the series. Select the correct answer.

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n+3}$$

a. $[-1, 1]$
b. $(-1, 1)$
c. $(-1, 1]$
d. diverges everywhere
e. $[-1, 1)$

16. Find the radius of convergence of the series.

$$\sum_{n=1}^{\infty} \frac{n^3 x^n}{2^n}$$

17. Find a power series representation for the function.

$$f(y) = \ln\left(\frac{11+y}{11-y}\right)$$

____ 18. Use the power series for $f(x) = \sqrt[3]{5+x}$ to estimate $\sqrt[3]{5.07}$ correct to four decimal places.

Select the correct answer.

a. 1.7179
b. 1.7189
c. 1.7195
d. 1.7156
e. 1.7200

19. Given the series $\sum_{m=1}^{\infty} \frac{3m}{4^m(3m+5)}$ estimate the error in using the partial sum s_8 by comparison with the series $\sum_{m=9}^{\infty} \frac{1}{4^m}$.

____ 20. Evaluate the function $f(x) = \cos x$ by a Taylor polynomial of degree 4 centered at $a = 0$, and $x = \frac{\pi}{4}$.

Select the correct answer.

a. 0.7074
b. 4.2074
c. 3.2074
d. 2.2074
e. 1.2074

<u>Stewart – Calculus ET 8e Chapter 11 Form H</u>

Answer Key

1. A
2. $789.56
3. Diverges
4. 36
5. B
6. $-\dfrac{1}{6}$
7. Diverges
8. C
9. $p > 1$
10. The series is convergent.
11. –0.90
12. 3.97036
13. B
14. $R = 0, I = \{0\}$
15. C
16. $R = 2$
17. $\displaystyle\sum_{n=0}^{\infty} \frac{2y^{2n+1}}{11^{n+1}(2n+1)}$
18. A
19. $R_8 \leq 0.0000051$
20. A